Seksjon 1.3

Komplekse Fourierrekker

$$\cos(2\pi nt/T) = \frac{1}{2}\left(e^{2\pi int/T} + e^{-2\pi int/T}\right)$$

$$\sin(2\pi nt/T) = \frac{1}{2i}\left(e^{2\pi int/T} - e^{-2\pi int/T}\right)$$

$$e^{2\pi int/T} = \cos 2\pi int/T + i\sin(2\pi nt/T)$$

Et av aksiomene i et komplekst indreproduktrom er:

$$\langle f, g \rangle = \overline{\langle g, f \rangle}$$

Får nå:

$$\langle f, cg \rangle = \overline{\langle cg, f \rangle} = \overline{c\langle g, f \rangle} = \bar{c}\,\overline{\langle g, f \rangle}$$

$$= \bar{c}\langle f, g \rangle$$

NB!

Oppgave 1.9: Vis at $\{e^{2\pi i n t/T}\}$ er en ortonormal basis.

$$f_N(t) = \sum_{n=-N}^{N} \frac{\langle f, e^{2\pi i n t/T} \rangle}{\langle e^{2\pi i n t/T}, e^{2\pi i n t/T} \rangle} e^{2\pi i n t/T}$$

(ort. dekomp teorem.

$$f_N(t) = \sum_{n=-N}^{N} \langle f, e^{2\pi i n t/T} \rangle e^{2\pi i n t/T}$$

$$= \sum_{n=-N}^{N} y_n e^{2\pi i n t/T}$$

1.2.5. eksempel.

Hva er den komplekse Fourierrekka til

$f(t) = \cos^3(2\pi t/3)$?

Her finnes en enkel metode:

periode $T=3$

$$\cos^3(2\pi t/3) = \left(\frac{1}{2}\left(e^{2\pi i t/3} + e^{-2\pi i t/T}\right)\right)^3$$

$$= \underbrace{\frac{1}{8}}_{y_3} \underbrace{e^{2\pi i 3t/3}}_{n=3} + \underbrace{\frac{3}{8}}_{y_1} \underbrace{e^{2\pi i t/3}}_{n=1} + \underbrace{\frac{3}{8}}_{y_{-1}} \underbrace{e^{-2\pi i t/3}}_{n=-1} + \underbrace{\frac{1}{8}}_{y_{-3}} \underbrace{e^{-2\pi i 3t/3}}_{n=-3}$$

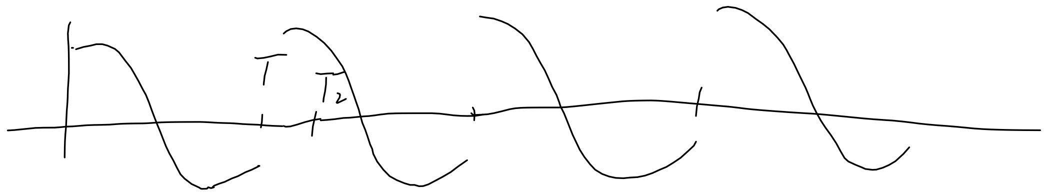
$$\sum_{n=-N}^{N} y_n e^{2\pi i n t/3}$$

$f_3(t)$ (fire ledd)

$$f_1(t) = \frac{3}{8} e^{2\pi i T/3} + \frac{3}{8} e^{-2\pi i t/3}$$

Eksempel 1.23

$f(t) = e^{2\pi i t/T_2}$ over $[0,T]$, der $T < T_2$



$$y_n = \frac{1}{T}\int_0^T f(t)\, e^{-2\pi i n t/T}\, dt$$

$$= \frac{1}{T}\int_0^T e^{2\pi i t/T_2}\, e^{-2\pi i n t/T}\, dt$$

$$= \frac{1}{T}\int_0^T e^{2\pi i t(1/T_2 - n/T)}\, dt$$

$$= \frac{1}{T}\,\frac{1}{2\pi i(1/T_2 - n/T)}\left[e^{2\pi i t(1/T_2 - n/T)}\right]_0^T$$

$$= \frac{1}{2\pi i(T/T_2 - n)}\left(e^{2\pi i(T/T_2 - n)} - 1\right) \qquad e^{2\pi i n} = 1$$

$$= \frac{1}{2\pi i(T/T_2 - n)}\left(e^{2\pi i T/T_2} - 1\right)$$