Fra fredag: kap 1.3

Oppg 1.12, 1.13

$$\frac{1}{T}\int_0^T f(t)\, e^{-2\pi i n t/T}\, dt = y_n$$

Bevis teorem 1.27, del 2

$$y_m = \frac{1}{T}\int_0^T f(t)\, e^{-2\pi i m t/T}\, dt$$

$$= \frac{1}{T}\int_0^T e^{2\pi i n t/T}\, e^{-2\pi i m t/T}\, dt$$

$$= \frac{1}{T}\int_0^T e^{2\pi i (n-m) t/T}\, dt$$

$$= \begin{cases} n = m: & \frac{1}{T}\cdot T = 1 \\ n \neq m: & \frac{1}{T}\frac{T}{2\pi i(n-m)}\left[e^{2\pi i (n-m) t/T}\right]_0^T \end{cases}$$

$$= \begin{cases} n = m & 1 \\ n \neq m: & \frac{1}{2\pi i(n-m)}\left(\overset{\nearrow 1}{e^{2\pi i (n-m)}} - 1\right) = 0 \end{cases}$$

Teorem 1.28, $f(t) = \overline{f(t)}$ $e^{-2\pi int/T} = \overline{e^{2\pi int/T}}$

bevis for at $y_n = \overline{y_{-n}}$ (for f reell)

$$y_n = \frac{1}{T}\int_0^T f(t) e^{-2\pi int/T} dt = \overline{\frac{1}{T}\int_0^T f(t) e^{2\pi int/T} dt}$$

$$= \overline{\frac{1}{T}\int_0^T f(t) e^{-2\pi i(-n)t/T} dt} \overset{\substack{\text{Formel}\\ \text{for } y_{-n}}}{=} \overline{y_{-n}}$$

Bevis for Fourierrekka til $g(t) = f(t-d)$:

$$\frac{1}{T}\int_0^T g(t) e^{-2\pi int/T} dt = \frac{1}{T}\int_0^T f(t-d) e^{-2\pi int/T} dt$$

$$\overset{u=t-d}{=} \frac{1}{T}\int_0^T f(u) e^{-2\pi in(u+d)/T} dt$$

$$= \frac{1}{T}\int_0^T f(u) e^{-2\pi inu/T} e^{-2\pi ind/T} du$$

$$= e^{-2\pi ind/T} \frac{1}{T}\int_0^T f(u) e^{-2\pi inu/T} du = \underline{\underline{e^{-2\pi ind/T} y_n}}$$

Starten av andre time: oppg 1.17

Lemma 1.29 Anta at f er deriverbar. Da er

$$(f_N)' = (f')_N$$

Bevis: $y_n = \langle f, e^{2\pi i n t/T} \rangle = \frac{1}{T}\int_0^T f(t) e^{-2\pi i n t/T} dt$

$$= \frac{1}{T}\Big(\underbrace{\Big[f(t)\Big(-\frac{T}{2\pi i n}\Big) e^{-2\pi i n t/T} \Big]_0^T}_{0 \text{ siden } f \text{ periodisk og } e^{-2\pi i n t/T} \text{ periodisk}} + \frac{T}{2\pi i n}\int_0^T f'(t) e^{-2\pi i n t/T} dt \Big)$$

$$= \frac{T}{2\pi i n}\,\frac{1}{T}\int_0^T f'(t) e^{-2\pi i n t/T} dt = \underline{\frac{T}{2\pi i n}\langle f', e^{2\pi i n t/T} \rangle}$$

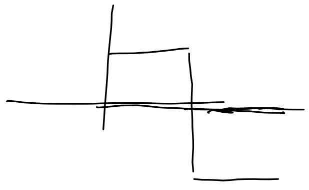
Vi får nå $(f_N)' = \Big(\sum_{n=-N}^{N} \langle f, e^{2\pi i n t/T} \rangle e^{2\pi i n t/T} \Big)'$

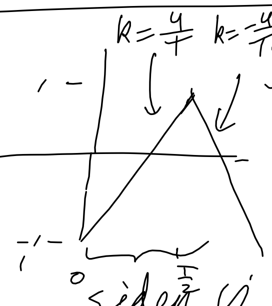
$$= \sum_{n=-N}^{N} \underline{\langle f, e^{2\pi i n t/T} \rangle \frac{2\pi i n}{T}}\, e^{2\pi i n t/T}$$

brukt det vi regnet ut over →

$$= \sum_{n=-N}^{N} \langle f', e^{2\pi i n t/T} \rangle e^{2\pi i n t/T}$$

$$= (f')_N$$





Ser at:

$$(f_t)' = \frac{4}{T} f_s$$

Vi har dermed f_t', siden vi allerede har f_s

$\Rightarrow$ Vi har allerede $(f_t')_N$, og dermed $((f_t)_N)'$

$\Rightarrow (f_t)_N$ kan finnes ved integrasjon.