prop 5.12

bevis

$f_{n,1}$ $\quad$ $f_{n,2}$

$\underbrace{[n, n+\tfrac{1}{2})}_{\phi_{1,2n} \neq 0 \text{ her}}$ $\quad$ $\underbrace{[n+\tfrac{1}{2}, n+1)}_{\phi_{1,2n+1} \neq \text{ her}}$

$$\Rightarrow f(t) = \sum_{n=0}^{N-1} \underbrace{f_{n,1}\tfrac{1}{\sqrt{2}}\phi_{1,2n}}_{[n,n+\frac{1}{2})} + \underbrace{f_{n,2}\tfrac{1}{\sqrt{2}}\phi_{1,2n+1}}_{[n+\frac{1}{2},n+1)}$$

$$\text{proj}_{V_0} f = \sum_{n=0}^{N-1} \langle f, \phi_{0,n}\rangle \phi_{0,n}$$

$$= \sum_{n=0}^{N-1} \left\langle f_{n,1}\tfrac{1}{\sqrt{2}}\phi_{1,2n} + f_{n,2}\tfrac{1}{\sqrt{2}}\phi_{1,2n+1}, \phi_{0,n}\right\rangle \phi_{0,n}$$

$$= \sum_{n=0}^{N-1} \left(\frac{f_{n,1}}{\sqrt{2}} \underbrace{\langle \phi_{1,2n}, \phi_{0,n}\rangle}_{\sqrt{2} \quad 1} + \frac{f_{n,2}}{\sqrt{2}} \underbrace{\langle \phi_{1,2n+1}, \phi_{0,n}\rangle}_{\sqrt{2} \quad 1 \;\; \frac{\sqrt{2}}{2}} \right) \phi_{0,n}$$

$$\int_n^{n+1} \phi_{1,2n}\phi_{0,n}\,dt = \sqrt{2}\cdot\tfrac{1}{2}$$

$$= \sum_{n=0}^{N-1} \left(\frac{f_{n,1}}{\sqrt{2}}\frac{\sqrt{2}}{2} + \frac{f_{n,2}}{\sqrt{2}}\frac{\sqrt{2}}{2}\right)\phi_{0,n} = \sum_{n=0}^{N-1} \frac{f_{n,1}+f_{n,2}}{2} \phi_{0,n}$$

$\Rightarrow \text{proj}_{V_0} f$ e $\dfrac{f_{n,1}+f_{n,2}}{2}$ på $[n, n+1)$

$$f_{n,1} - \frac{f_{n,1}+f_{n,2}}{2} = \frac{f_{n,1}-f_{n,2}}{2}$$

$\text{proj}_{W_0} f = f - \text{proj}_{V_0} f$

$$f_{n,2} - \frac{f_{n,1}+f_{n,2}}{2} = \frac{f_{n,2}-f_{n,1}}{2}$$

$$\phi_{0,n} = \frac{1}{\sqrt{2}}\phi_{1,2n} + \frac{1}{\sqrt{2}}\phi_{1,2n+1} \qquad \times \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\psi_{0,n} = \frac{1}{\sqrt{2}}\phi_{1,2n} - \frac{1}{\sqrt{2}}\phi_{1,2n+1}$$

(er sin egen invers)

$[n, n+1) \qquad [n, n+\frac{1}{2}) \cup [n+\frac{1}{2}, n)$

Oppgave 5.18 og 5.19