Fra sist: Viste:

$$\phi_{0,n} = \frac{1}{\sqrt{2}}\left(\frac{1}{2}\phi_{1,2n+1} + \phi_{1,2n} + \frac{1}{2}\phi_{1,2n-1}\right)$$

$$\psi_{0,n} = \frac{1}{\sqrt{2}}\phi_{1,2n+1}$$

Dette gir oss søylene i koordinatskifte-matrisen: $(\phi_{0,n}, \psi_{0,n})_n \longrightarrow \{\phi_{1,n}\}_n$, det vil si en IDWT

Motsatt vei: Flytter over

$$\phi_{1,2n} = \sqrt{2}\phi_{0,n} - \frac{1}{2}\phi_{1,2n+1} - \frac{1}{2}\phi_{1,2n-1}$$

$$\phi_{1,2n+1} = \sqrt{2}\psi_{0,n}$$

$$\phi_{1,2n} = \sqrt{2}\phi_{0,n} - \frac{1}{2}\sqrt{2}\psi_{0,n} - \frac{1}{2}\sqrt{2}\psi_{0,n-1}$$

$$= \sqrt{2}\left(-\frac{1}{2}\psi_{0,n-1} + \phi_{0,n} - \frac{1}{2}\psi_{0,n}\right)$$

Dette gir oss DWT-matrisen.

Kap. 6:

Vi har sett:

For Haar-wavelet:

$P_{C_m \leftarrow \phi_m}$: $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$ repeteres på diagonalen

For stykkevis lineær wavelet:

$P_{\phi_m \leftarrow C_m}$: $\sqrt{2}\begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & \vdots \\ \frac{1}{2} & 0 \end{pmatrix}$ blir repetert/forskjøvet.

$P_{C_m \leftarrow \phi_m}$: $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & \\ -\frac{1}{2} & 0 \end{pmatrix}$ blir repetert/forskjøvet.

Ser at, i alle matrisene, blir annenhver søyle repetert, akkurat som i et filter.

Bevis for teorem 6.5

$$c_m = G\begin{pmatrix} c_{m-1,0} \\ w_{m-1,0} \\ c_{m-1,1} \\ w_{m-1,1} \\ \vdots \end{pmatrix} = G\left(\begin{pmatrix} c_{m-1,0} \\ 0 \\ c_{m-1,1} \\ 0 \\ \vdots \end{pmatrix} + \begin{pmatrix} 0 \\ w_{m-1,0} \\ 0 \\ w_{m-1,1} \\ \vdots \end{pmatrix}\right)$$

$$= G\begin{pmatrix} c_{m-1,0} \\ 0 \\ c_{m-1,1} \\ 0 \\ \vdots \end{pmatrix} + G\begin{pmatrix} 0 \\ w_{m-1,0} \\ 0 \\ w_{m-1,1} \\ \vdots \end{pmatrix} = G_0\begin{pmatrix} c_{m-1,0} \\ 0 \\ c_{m-1,1} \\ 0 \\ \vdots \end{pmatrix} + G_1\begin{pmatrix} 0 \\ w_{m-1,0} \\ 0 \\ w_{m-1,1} \\ \vdots \end{pmatrix}$$

Hva blir H_0, H_1, G_0, G_1 for Haar wavelets?

$$H = \left(\begin{array}{cc|cc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \multicolumn{2}{c}{0} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & & \\ \hline \multicolumn{2}{c|}{0} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ & & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ & & & \ddots \end{array}\right) = G$$

$$t_{-1} \quad t_0$$

$$H_0: \begin{array}{c} 0 \\ 1 \\ 2 \end{array} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \cdots \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \vdots & & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & & & \\ \frac{1}{\sqrt{2}} & & & \end{pmatrix} \qquad H_0 = \left\{\frac{1}{\sqrt{2}}, \underline{\frac{1}{\sqrt{2}}}\right\}$$

$$H_1: \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 & \cdots & \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \cdot & \cdot & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad H_1 = \left\{\underline{-\frac{1}{\sqrt{2}}}, \frac{1}{\sqrt{2}}\right\}$$

$$G_0: \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad G_0 = \left\{\underline{\frac{1}{\sqrt{2}}}, \frac{1}{\sqrt{2}}\right\}$$

søyle 0

søyle 1

$$G_1: \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} \\ \vdots & 0 \\ 0 & \vdots \\ \frac{1}{\sqrt{2}} & \end{pmatrix} \qquad G_1 = \left\{\frac{1}{\sqrt{2}}, \underline{-\frac{1}{\sqrt{2}}}\right\}$$

$$\lambda_{G_0}(\omega) = \frac{1}{\sqrt{2}}\left(1 + e^{-i\omega}\right)$$

$$\lambda_{G_1}(\omega) = \frac{1}{\sqrt{2}}\left(e^{i\omega} - 1\right)$$

$$\lambda_{H_0}(\omega) = \frac{1}{\sqrt{2}}\left(e^{i\omega} + 1\right)$$

$$\lambda_{H_1}(\omega) = \frac{1}{\sqrt{2}}\left(-1 + e^{-i\omega}\right)$$