Bevis for teorem 9,14:

Anvend først S_1 på søylene i X $(X \to Y)$
for hver j:

$$Y_{i,j} = \sum_{k_1} (t_1)_{k_1} X_{i-k_1, j}$$

Anvend så S_2 på radene i Y $(Y \to Z)$
for hver i:

$$Z_{i,j} = \sum_{k_2} (t_2)_{k_2} Y_{i, j-k_2}$$

$$= \sum_{k_2} (t_2)_{k_2} \sum_{k_1} (t_1)_{k_1} X_{i-k_1, j-k_2}$$

$$= \sum_{k_1, k_2} \underbrace{(t_1)_{k_1} (t_2)_{k_2}}_{a_{k_1, k_2}} X_{i-k_1, j-k_2}$$

Bevis for teorem 9.19:

$$(S_1 \otimes S_2)(e_i \otimes e_j) \overset{\text{def}}{=} (S_1 e_i) \otimes (S_2 e_j)$$

$$\overset{\text{def}}{=} (S_1 e_i)(S_2 e_j)^T$$

$$= S_1 e_i e_j^T S_2^T$$

$$= S_1 (e_i \otimes e_j) S_2^T$$

$\Rightarrow (S_1 \otimes S_2) X = S_1 X S_2^T$ for alle X på formen $e_i \otimes e_j$

$\Rightarrow (S_1 \otimes S_2) X = S_1 X S_2^T$ for alle X, siden $e_i \otimes e_j$ er basis for alle matriser.