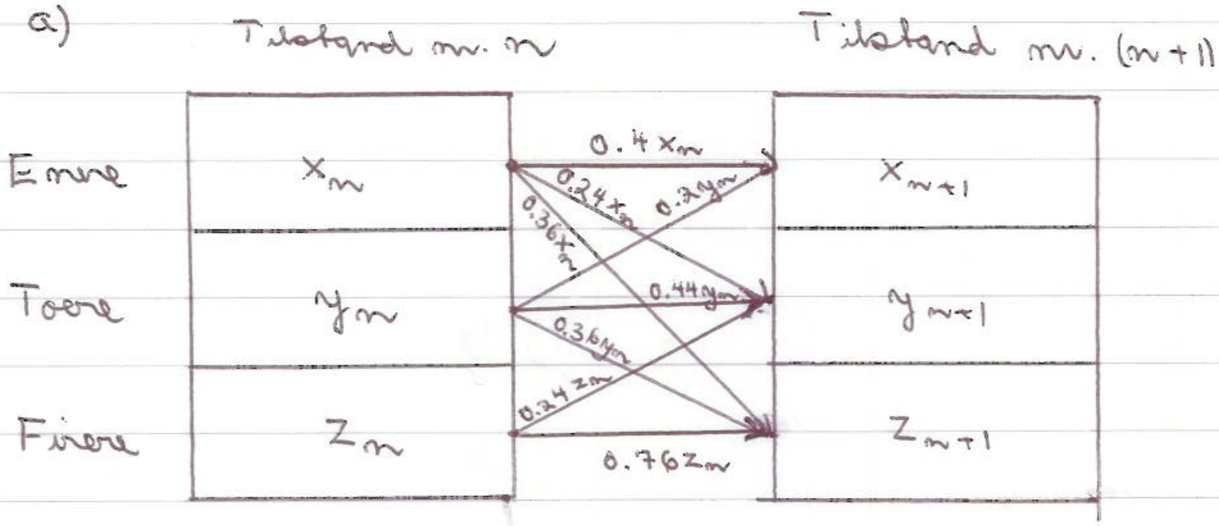
FASIT - OBLIG 1

1

a)



Vi summerer bidragene:

$$x_{n+1} = 0.4\,x_n + 0.2\,y_n$$

$$y_{n+1} = 0.24\,x_n + 0.44\,y_n + 0.24\,z_n$$

$$z_{n+1} = 0.36\,x_n + 0.36\,y_n + 0.76\,z_n$$

Det var dette vi skulle vise.

b) $$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} 0.4 & 0.2 & 0 \\ 0.24 & 0.44 & 0.24 \\ 0.36 & 0.36 & 0.76 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 100 \end{pmatrix} = \begin{pmatrix} 0 \\ 24 \\ 76 \end{pmatrix}$$

Andre trekning: 76 i firere, 24 i toere, ingen i enere.

$$\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix} = \begin{pmatrix} 0.4 & 0.2 & 0 \\ 0.24 & 0.44 & 0.24 \\ 0.36 & 0.36 & 0.76 \end{pmatrix} \begin{pmatrix} 0 \\ 24 \\ 76 \end{pmatrix} = \begin{pmatrix} 4.8 \\ 28.8 \\ 66.4 \end{pmatrix}$$

Tredje trekning: 66 i firere, 29 i toere, 5 i enere.

c) Karakteristisk ligning:

$$\det(M-\lambda I) = \begin{vmatrix} 0.4-\lambda & 0.2 & 0 \\ 0.24 & 0.44-\lambda & 0.24 \\ 0.36 & 0.36 & 0.76-\lambda \end{vmatrix}$$

$$= (0.4-\lambda)\begin{vmatrix} 0.44-\lambda & 0.24 \\ 0.36 & 0.76-\lambda \end{vmatrix} - 0.2\begin{vmatrix} 0.24 & 0.24 \\ 0.36 & 0.76-\lambda \end{vmatrix}$$

$$= (0.4-\lambda)\Big((0.44-\lambda)(0.76-\lambda) - 0.36\cdot 0.24\Big) - 0.2\Big(0.24\cdot(0.76-\lambda) - 0.36\cdot 0.24\Big)$$

$$= (0.4-\lambda)\left(0.3344 - 0.44\lambda - 0.76\lambda + \lambda^2 - 0.0864\right) - 0.2\left(0.1824 - 0.24\lambda - 0.0864\right)$$

$$= (0.4-\lambda)(\lambda^2 - 1.2\lambda + 0.248) - 0.2(0.096 - 0.24\lambda)$$

$$= 0.4\lambda^2 - 0.48\lambda + 0.0992 - \lambda^3 + 1.2\lambda^2 - 0.248\lambda - 0.0192 + 0.048\lambda$$

$$= \underline{\underline{-\lambda^3 + 1.6\lambda^2 - 0.68\lambda + 0.08 = 0.}}$$

$$p(\lambda) = -\lambda^3 + 1.6\lambda^2 - 0.68\lambda + 0.08$$

$$p(1) = -1 + 1.6 - 0.68 + 0.08 = 0$$

$$p(0.2) = -0.008 + 1.6\cdot 0.04 - 0.68\cdot 0.2 + 0.08$$

$$= -0.008 + 0.064 - 0.136 + 0.080 = 0$$

$$p(0.4) = -0.064 + 1.6\cdot 0.16 - 0.68\cdot 0.4 + 0.08$$

$$= -0.064 + 0.256 - 0.272 + 0.080 = 0.$$

d) Egenvektorer

$\lambda_1 = 1$. Ligningen:

$$\left[\begin{array}{cccc} -0.6 & 0.2 & 0 & 0 \\ 0.24 & -0.56 & 0.24 & 0 \\ \cancel{0.36} & \cancel{0.36} & \cancel{-0.24} & \cancel{0} \end{array}\right] \underset{R_3}{\overset{\text{Stryker}}{\sim}} \left[\begin{array}{cccc} -0.6 & 0.2 & 0 & 0 \\ 0.24 & -0.56 & 0.24 & 0 \end{array}\right]$$

$$\overset{-\frac{1}{0.6}R_1}{\sim} \left[\begin{array}{cccc} 1 & -\frac{1}{3} & 0 & 0 \\ 0.24 & -0.56 & 0.24 & 0 \end{array}\right] \overset{-0.24R_1 \text{ til } R_2}{\sim} \left[\begin{array}{cccc} 1 & -\frac{1}{3} & 0 & 0 \\ 0 & -0.48 & 0.24 & 0 \end{array}\right]$$

$$\overset{-\frac{1}{0.48}R_2}{\sim} \left[\begin{array}{cccc} 1 & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \end{array}\right]$$

z velges fritt.

$z = s$

$y - \frac{1}{2}z = 0 \Rightarrow y = \frac{1}{2}z = \frac{1}{2}s$

$x - \frac{1}{3}y = 0 \Rightarrow x = \frac{1}{3}y = \frac{1}{6}s$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s \begin{bmatrix} \frac{1}{6} \\ \frac{1}{2} \\ 1 \end{bmatrix}$$ Ganger med 6: Egenvektorer $\underline{\underline{s\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}}}$, $s \in \mathbb{R}$

$\lambda_2 = 0.2$. Ligningen:

$$\left[\begin{array}{cccc} 0.2 & 0.2 & 0 & 0 \\ 0.24 & 0.24 & 0.24 & 0 \\ \cancel{0.36} & \cancel{0.36} & \cancel{0.56} & \cancel{0} \end{array}\right] \underset{R_3}{\overset{\text{Stryker}}{\sim}} \left[\begin{array}{cccc} 0.2 & 0.2 & 0 & 0 \\ 0.24 & 0.24 & 0.24 & 0 \end{array}\right]$$

$$\overset{\frac{1}{0.2}R_1 \text{ og } \frac{1}{0.24}R_2}{\sim} \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \overset{-R_1 \text{ til } R_2}{\sim} \left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

y velges fritt

$z = 0$, $y = t$, $x + y = 0 \Rightarrow x = -y = -t$

$$\underline{\underline{t\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}}}, \quad t \in \mathbb{R}.$$

$\lambda_3 = 0.4$. Ligningen:

$$\begin{bmatrix} 0 & 0.2 & 0 & 0 \\ \cancel{0.24} & \cancel{0.04} & \cancel{0.24} & \cancel{0} \\ 0.36 & 0.36 & 0.36 & 0 \end{bmatrix} \underset{\text{Bytter } R_1 \text{ og } R_3}{\overset{\text{Stryker } R_2.}{\sim}} \begin{bmatrix} 0.36 & 0.36 & 0.36 & 0 \\ 0 & 0.2 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{0.36}R_1 \text{ og } \frac{1}{0.2}R_2 \sim \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

z velges fritt.

$z = u$, $y = 0$, $x + y + z = 0 \Rightarrow x = -y - z = -u$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = u \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad u \in \mathbb{R}.$$

e) Vi skriver u_1 som en sum av egenvektoren

$$s\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} + t\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + u\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix}$$ som gir systemet

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 3 & 1 & 0 & 0 \\ 6 & 0 & 1 & 100 \end{bmatrix} \underset{-6R_1 \text{ til } R_3}{\overset{-3R_1 \text{ til } R_2}{\sim}} \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 4 & 3 & 0 \\ 0 & 6 & 7 & 100 \end{bmatrix}$$

$$\frac{1}{4}R_2 \sim \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0.75 & 0 \\ 0 & 6 & 7 & 100 \end{bmatrix} \overset{-6R_2 \text{ til } R_3}{\sim} \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & 0.75 & 0 \\ 0 & 0 & 2.5 & 100 \end{bmatrix}$$

$2.5u = 100$, $u = \frac{100}{2.5} = 40$

$t + 0.75u = 0 \Rightarrow t = -0.75u = -30$

$s - t - u = 0 \Rightarrow s = t + u = 40 - 30 = 10$

Vi har

$$\begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} = M^{n-1} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = M^{n-1} \left(10 \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} - 30 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + 40 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right)$$

$$= 1^{n-1} \cdot 10 \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} - 0.2^{n-1} \cdot 30 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + 0.4^{n-1} \cdot 40 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$x_n = 10 + 30 \cdot 0.2^{n-1} - 40 \cdot 0.4^{n-1}$$

$$y_n = 30 - 30 \cdot 0.2^{n-1}$$

$$z_n = 60 + 40 \cdot 0.4^{n-1}$$

Når n går mot uendelig, vil x_n gå mot 10, y_n mot 30 og z_n mot 60.

2

a) $\begin{vmatrix} 1 & 2 & a \\ a & -1 & 3 \\ 2 & 5 & 0 \end{vmatrix} = 1 \cdot \begin{vmatrix} -1 & 3 \\ 5 & 0 \end{vmatrix} - 2 \begin{vmatrix} a & 3 \\ 2 & 0 \end{vmatrix} + a \begin{vmatrix} a & -1 \\ 2 & 5 \end{vmatrix}$

$= 1 \cdot (0-15) - 2(0-6) + a(5a-(-2))$

$= -15 + 12 + 5a^2 + 2a = \underline{\underline{5a^2 + 2a - 3}}$

b) Ved at vi har en lösning hvis og bare hvis det $A \neq 0$.

$5a^2 + 2a - 3 = 0 \Rightarrow a = \dfrac{-2 \pm \sqrt{4+60}}{10} = \dfrac{-2 \pm 8}{10} = \begin{cases} -1 \\ 0.6 \end{cases}$

Altså: En lösning når $a \neq -1$ og $a \neq 0.6$.

Må sjekke -1 og 0.6 :

-1 :

$\begin{bmatrix} 1 & 2 & -1 & 1 \\ -1 & -1 & 3 & 4 \\ 2 & 5 & 0 & 7 \end{bmatrix} \underset{\sim}{\overset{+R_1 \text{ til } R_2,\ -2R_1 \text{ til } R_3}{}} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 2 & 5 \end{bmatrix}$

$\underset{\sim}{-R_2 \text{ til } R_3} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Har 1 fri variabel, altså uendelig mange lösninger.

0.6 :

$$\begin{bmatrix} 1 & 2 & 0.6 & 1 \\ 0.6 & -1 & 3 & 4 \\ 2 & 5 & 0 & 7 \end{bmatrix} \underset{\sim}{\overset{-0.6R_1 \text{ til } R_2}{-2R_1 \text{ til } R_3}} \begin{bmatrix} 1 & 2 & 0.6 & 1 \\ 0 & -2.2 & 2.64 & 3.4 \\ 0 & 1 & -1.2 & 5 \end{bmatrix}$$

$$\underset{\sim}{\text{Bytter } R_2 \text{ og } R_3} \begin{bmatrix} 1 & 2 & 0.6 & 1 \\ 0 & 1 & -1.2 & 5 \\ 0 & -2.2 & 2.64 & 3.4 \end{bmatrix} \underset{\sim}{\overset{+2.2R_2}{\text{til } R_3}} \begin{bmatrix} 1 & 2 & 0.6 & 1 \\ 0 & 1 & -1.2 & 5 \\ 0 & 0 & 0 & 14.4 \end{bmatrix}$$

Siste ligning er $0 = 14.4$. Umulig.

Altså ingen løsning.