FASIT-OBLIG 2.

1. $x_{n+2} + x_{n+1} - 2x_n = 6n + 11$

a) Karakteristisk ligning: $r^2 + r - 2 = 0$

$$r = \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = \begin{cases} 1 \\ -2 \end{cases}$$

$$x_n^h = C \cdot 1^n + D(-2)^n = \underline{\underline{C + D(-2)^n}}$$

b) Siden 1 er rot i den karakteristiske ligningen, må vi velge x_n som et polynom av grad 1 mer enn $6n+10$, dvs.

$$x_n = An^2 + Bn$$

$$x_{n+2} + x_{n+1} - 2x_n = A(n+2)^2 + B(n+2) + A(n+1)^2 + B(n+1)$$

$$-2An^2 - 2Bn = A(n^2+4n+4) + B(n+2) + A(n^2+2n+1)$$

$$+ B(n+1) - 2An^2 - 2Bn =$$

$$n^2(A + A - 2A) + n(4A + B + 2A + B - 2B) + (4A + 2B + A + B)$$

$$= n(6A) + (5A + 3B)$$

Får $6A = 6 \Rightarrow A = 1$

$5A + 3B = 11 \Rightarrow 3B = 6$, $B = 2$

$$x_n^p = n^2 + 2n$$

Generell løsning av den inhomogene

$$x_n = x_n^h + x_n^p = C + D(-2)^n + n^2 + 2n$$

c) $\left.\begin{aligned} x_0 &= C + D = 2 \\ x_1 &= C - 2D + 1 + 2 = 2 \\ & C - 2D = -1 \end{aligned}\right\} \; C = D = 1$

$$\underline{\underline{x_n = 1 + (-2)^n + n^2 + 2n.}}$$

2.

a) $$\frac{x-2}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$x - 2 = A(x-1) + Bx = (A+B)x - A$$

$$A = 2$$
$$A + B = 1 \Rightarrow B = -1.$$

$$\int \frac{x-2}{x(x-1)}\,dx = \int \frac{2}{x} - \frac{1}{x-1}\,dx = 2\ln|x| - \ln|x-1| + C$$

$$\underline{\underline{= \ln \frac{x^2}{|x-1|} + C.}}$$

b) $x(x-1)y' + (x-2)y = 2(x-1)^2$. Antar $x \neq 0,\ x \neq 1$.

$$y' + \frac{x-2}{x(x-1)}\,y = \frac{2(x-1)}{x}$$

Integrerende faktor $m(x) = e^{\int \frac{x-2}{x(x-1)}\,dx} = e^{\ln \frac{x^2}{|x-1|}} = \frac{x^2}{|x-1|}$

Kan velge $m(x) = \dfrac{x^2}{(x-1)}$

Ligningen blir

$$\left(\frac{x^2}{(x-1)}\,y\right)' = \frac{x^2}{(x-1)}\cdot\frac{2(x-1)}{x} = 2x$$

$$\frac{x^2}{x-1}\,y = x^2 + C$$

$$\underline{\underline{y = \frac{(x^2+C)(x-1)}{x^2}}}$$

Ser at denne passer også når $x=1$. For at den skal være definert også for $x=0$, må vi ha $C=0$, dvs

$$\underline{\underline{y = x-1.}}$$

c) $y(2) = \dfrac{4+C}{4} = 2$; dvs $C=4$.

$$\underline{\underline{y = \frac{(x^2+4)(x-1)}{x^2}}}$$

$\underline{\underline{3.}}$

a) $\displaystyle\int \frac{3x^2}{2\sqrt{x^3+1}}\,dx = \int \frac{du}{2\sqrt{u}} = \int \frac{1}{2}u^{-\frac{1}{2}}\,du = \frac{1}{2}\cdot 2u^{\frac{1}{2}} + C$

$u = x^3+1$

$du = 3x^2dx$

$$= u^{\frac{1}{2}} + C = \underline{\underline{\sqrt{x^3+1} + C.}}$$

b) $2y'\sqrt{x^3+1} + 3x^2(y-1)^2 = 0$

$$2\sqrt{x^3+1}\,\frac{dy}{dx} = -3x^2(y-1)^2$$

$$-\frac{dy}{(y-1)^2} = \frac{3x^2\,dx}{2\sqrt{x^3+1}}$$ Integrerer hver side!

V.S. $-\int \frac{dy}{(y-1)^2} = \frac{1}{y-1}$

H.S Fra oppgave a) : $\sqrt{x^3+1} + C$

$$\frac{1}{y-1} = \sqrt{x^3+1} + C$$

$$y-1 = \frac{1}{\sqrt{x^3+1}+C}$$

$$y = 1 + \frac{1}{\sqrt{x^3+1}+C}$$

$$y(2) = 1 + \frac{1}{\sqrt{8+1}+C} = 1 + \frac{1}{3+C} = 5/4$$

$$\frac{1}{3+C} = \frac{1}{4} \qquad C = 1.$$

$$\underline{\underline{y = 1 + \frac{1}{\sqrt{x^3+1}+1}}}$$

4. a) $\int \ln(x^2-1)\,dx = \int 1 \cdot \ln(x^2-1)\,dx =$

$$\boxed{\begin{array}{ll} u = \ln(x^2-1) & , u' = \dfrac{2x}{x^2-1} \\ v' = 1 & , v = x \end{array}}$$

$$x\ln(x^2-1) - \int \frac{2x^2}{x^2-1}\,dx$$

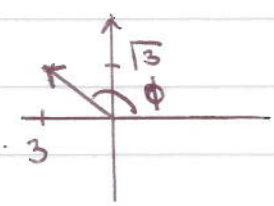
$$= x\ln(x^2-1) - \int \frac{2(x^2-1)+2}{x^2-1}\,dx = x\ln(x^2-1) - \int 2 + \frac{2}{x^2-1}\,dx$$

$$= x\ln(x^2-1) - 2x - \int \frac{2}{x^2-1}\,dx = x\ln(x^2-1) - 2x - \int \frac{1}{x-1} - \frac{1}{x+1}\,dx$$

$$= x\ln(x^2-1) - 2x - \ln|x-1| + \ln|x+1| + C$$

$$= x\ln(x^2-1) - 2x + \ln\left|\frac{x+1}{x-1}\right| + C.$$

b) $A = \sqrt{(-3)^2 + (\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$



$\cos\phi = \dfrac{-3}{2\sqrt{3}} = -\frac{1}{2}\sqrt{3}$, $\phi = \dfrac{5\pi}{6}$

$$f(x) = -3\cos 3x + \sqrt{3}\sin 3x = 2\sqrt{3}\cos\left(3x - \frac{5\pi}{6}\right)$$

$$= 2\sqrt{3}\cos 3\left(x - \frac{5\pi}{18}\right)$$

Amplitude $A = 2\sqrt{3}$

Periode $T = \dfrac{2\pi}{\omega} = \dfrac{2}{3}\pi$

Akrofase $\dfrac{5}{18}\pi$.