Fasit midtveiseksamen : B B C A C B C A D B D.

1) På matriseform

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 3 & -2 & 1 & a \\ 2 & 1 & 1 & 3a+3 \end{bmatrix} \overset{\substack{-3R_1 \text{ til } R_2 \\ -2R_1 \text{ til } R_3}}{\sim} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -2 & a-3 \\ 0 & 3 & -1 & 3a+1 \end{bmatrix}$$

$$\overset{-3R_2 \text{ til } R_3}{\sim} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -2 & a-3 \\ 0 & 0 & 5 & 10 \end{bmatrix}$$

$$5z = 10 \Rightarrow z = 2$$

$$y - 2z = a - 3 \Rightarrow y = a - 3 + 2z = a - 3 + 4 = \underline{\underline{a+1}}$$ B

2) Koeffisientmatrisen må ha determinant 0 :

$$\begin{vmatrix} a & 1 \\ 4 & a \end{vmatrix} = a^2 - 4 = 0 \iff a = \pm 2$$

For $a = +2$ er systemet

$$2x + y = 1$$
$$4x + 2y = 2$$

Dem er samme ligning, altså uendelig mange løsninger.

Svar : B

($a = -2$ gir ingen løsninger).

3) $$\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -a \\ 3 & -2 & 1 \end{vmatrix} = 1\cdot\begin{vmatrix} 1 & -a \\ -2 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -a \\ 3 & 1 \end{vmatrix} + 1\cdot\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix}$$

$$= 1\cdot(1-2a) + 1\cdot(1+3a) + 1\cdot(-2-3) = \underline{\underline{a-3}}$$ (C)

4) Egenverdier:

$$|M-\lambda I| = \begin{vmatrix} 3-\lambda & 8 \\ 1 & 5-\lambda \end{vmatrix} = (3-\lambda)(5-\lambda) - 8$$

$$= 15 - 3\lambda - 5\lambda + \lambda^2 - 8 = \lambda^2 - 8\lambda + 7 = 0$$

$$\lambda = \frac{8 \pm \sqrt{64-28}}{2} = \frac{8 \pm 6}{2} = \begin{cases} 7 \\ 1 \end{cases}$$

Svar: (A)

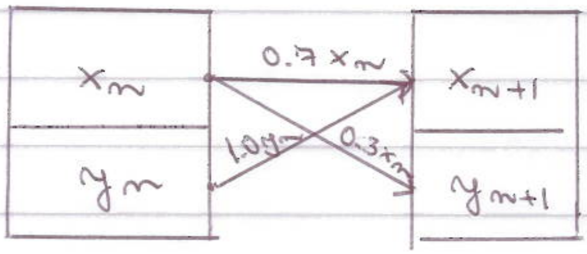
5)

$$\begin{array}{ccc|c|c|c|} & & & 0 & -1 & 1 \\ & & & 1 & 1 & -1 \\ & & & 0 & 1 & 2 \\ \hline 3 & 2 & 4 & 2 & 3 & 9 \\ -2 & 5 & -1 & & 6 & -9 \\ 8 & -4 & 3 & & & 18 \\ & & & \text{UMULIG.} & \text{NEI} & \end{array}$$

Ser at $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ er egenvektor med egenverdi 9.

Svar: (C)

6)



$$x_{n+1} = 0.7 x_n + 1.0 y_n$$
$$y_{n+1} = 0.3 x_n$$

Matrisen $\begin{bmatrix} 0.7 & 1.0 \\ 0.3 & 0 \end{bmatrix}$ Svar: B

7) $3(a+2i)(-1+i) + (7+3ai) =$

$3(-a+ai-2i-2) + (7+3ai) =$

$-3a+3ai-6i-6+7+3ai =$

$-3a+1-6i+6ai$

Realdel: $\underline{\underline{-3a+1}}$ Svar: C

8) $z = 2e^{i\frac{\pi}{2}} \cdot e^{i\frac{\pi}{6}} + 1 = 2e^{i\left(\frac{\pi}{2}+\frac{\pi}{6}\right)} + 1 = 2e^{i\frac{2\pi}{3}} + 1$

$= 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) + 1 = 2\left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}\,i\right) + 1$

$= -1 + \sqrt{3}\,i + 1 = \underline{\underline{\sqrt{3}\,i.}}$ Svar: A

9) Lösning homogen ligning: $x_n = C \cdot (0.6)^n$

Partikulär lösning: $x_n = A$. $A - 0.6A = 2 \Rightarrow A = 5$

Svar: D

10) Karakteristisk ligning

$$r^2 - 0.4r - 0.6 = 0$$

$$r = \frac{0.4 \pm \sqrt{0.16 + 2.4}}{2} = \frac{0.4 \pm 1.6}{2} = \begin{cases} 1 \\ -0.6 \end{cases}$$

Generell løsning $x_n = C \cdot 1^n + D(-0.6)^n = C + D(-0.6)^n$

$$\left.\begin{array}{l} x_0 = C + D = 50 \\ x_1 = C - 0.6D = 34 \end{array}\right. \Rightarrow \begin{array}{l} C = 40 \\ D = 10 \end{array}$$

x_n går mot $C = 40$. Svar: B

11) Karakteristisk ligning

$$r^2 - \sqrt{3}\, r + 1 = 0$$

$$r = \frac{\sqrt{3} \pm \sqrt{3-4}}{2} = \frac{1}{2}\sqrt{3} \pm \frac{1}{2} i$$

$$r = \frac{1}{2}\sqrt{3} + \frac{1}{2} i = e^{i\frac{\pi}{6}} \qquad (\rho = 1).$$

Generell løsning $x_n = C \cos n\frac{\pi}{6} + D \sin n\frac{\pi}{6}$

$$x_0 = C = 0$$
$$x_1 = D \sin \frac{\pi}{6} = D \cdot \frac{1}{2} = 1, \quad D = 2$$

$$x_{15} = 2 \sin \frac{15}{6}\pi = 2 \sin \frac{5}{2}\pi = 2 \sin \frac{1}{2}\pi = \underline{\underline{2}}$$

Svar: D