UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Examination in:	MAT3510/4510 — Geometric structures.
Day of examination:	Friday, December 1, 2006.
Examination hours:	14.30 – 17.30.
This examination set consists of 2 pages.	
Appendices:	None.
Permitted aids:	None.

Make sure that your copy of the examination set is complete before you start solving the problems.

Note: You must give reasons for all your answers. But you may use earlier results also if you have not been able to prove them.

Problem 1.

Compute the Gaussian curvature of the "helicoid", parametrized by

$$r(u,v) = (u\cos v, u\sin v, v)\,,\ u \in (0,\infty),\ v \in \mathbb{R}\,.$$

Problem 2.

(a) Consider the horizontal lines $l_b = \{x+iy \,|\, y = b\}$ in the hyperbolic upper half–plane $\mathbb{H}$, oriented "from left to right".

Find the parametrization by arc length $\alpha(s)$ of l_b such that $\alpha(0) = bi$.

(b) Explain why the geodesic curvature of l_b constant? Why is it also independent of b?

(c) Use the Gauss–Bonnet theorem on a suitable region bounded by a piece of some l_b and a geodesic segment to compute the geodesic curvature of l_b.

For which other curves in $\mathbb{H}$ and $\mathbb{D}$ can you now conclude that you have computed the geodesic curvature?

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Problem 3.

(a) Let $\mathbb{H}$ be the upper half–plane model for the hyperbolic plane, and let M be the subset $\{z \in \mathbb{H} \,|\, \operatorname{Im} z > 1\}$. Let $\mathcal{U} = \mathbb{R} \times (0, \pi/2)$ and define $x : \mathcal{U} \to M$ by $x(u, v) = u + \dfrac{i}{\cos v}$.

Show that x is a parametrization of M and compute the metric $ds^2 = E\,du^2 + 2F\,du\,dv + G\,du^2$ in these coordinates.

(b) Define $y : \mathcal{U} \to \mathbb{R}^3$ by

$$y(u, v) = (\cos v \cos u,\ \cos v \sin u,\ \sin v + \ln(\frac{1 - \sin v}{\cos v}))\,.$$

Show that y parametrizes a regular surface of rotation Σ in $\mathbb{R}^3$ and compute its metric in these coordinates.

(c) Show that there is an element γ generating an infinite cyclic subgroup $\Gamma \subset \mathit{M\ddot{o}b}(\mathbb{H})$ such that M/Γ and Σ are isometric.

Explain why this shows that Σ is a hyperbolic surface. What is its Gaussian curvature?

(d) Is Σ geodesically complete?

Let w be an arbitrary point in Σ. Explain how we can find infinitely many geodesics, each of which goes through w twice. Can any geodesic go through w more than two times?

(The surface Σ is often called the "pseudosphere".)

END