

# MAT4730/9730

## Mathematical Finance

### EXAM

To find out about date and place of the exam, please consult the webpage of the course or ask to the administrative support staff.

The exam is held at the blackboard for a duration of about 30-40 minutes in all.

A selection of topics from the list here below will be asked.

The candidate can organize his/her reply bearing in mind:

- to center the topic and address the major results
- to be prepared on the proofs of the statements, as presented/discussed in class or left as exercise after discussion
- to be prepared on the notions that are embedded in the topic addressed.

In addition, the candidate can be asked to use the notions and techniques acquired in some specific context, e.g. short exercise.

Hereafter is a list of topics which are going to be tested during the exam.

### TOPICS

#### Stochastic calculus

- Extended Itô integration and local-martingales compared with classical Itô integration
- Doob-Meyer decomposition theorem (statement)
- Martingale and local-martingale representation theorems

#### Single agent consumption/investment optimization

- Market model framework with Brownian noise, major stochastic processes of interest
- Portfolios: self-financing and  $\Gamma$ -financing
- Single agent consumption/investment scheme: utility functions and the preference structure, the optimization problem, admissibility, budget constrain
- The use of budget constrain in the replication result
- From the original single agent optimization problem to the auxiliary problem and return
- Solution of the auxiliary problem and Lagrange multipliers method

#### Pricing in an incomplete market due to constrains on portfolios

- Portfolios: constrains, admissibility, budget constrain
- Upper-hedging price
- Representation of the convex constrain on portfolios with support functions
- From an incomplete market with convex constrains on portfolios to a family of auxiliary markets and return

- Characterization of the upper-hedging price in general and in the case of European options with payoff function of polynomial growth

### **Single agent consumption/investment optimization with constraints on the portfolios**

- The optimization problem with constraints on portfolios: admissibility
- From the original problem to a family of constraint-free problems and return

### **Equilibrium in a complete market**

- The problem of equilibrium, the agents, the optimization of each agent (continuous endowment, consumption, portfolios, admissibility, budget constraints, feasibility constraints, preference structure, the single agent optimization problem)
- Solution of the single agent optimization problem
- Definition of equilibrium market
- Characterization of the equilibrium
- Representative agent and its utility function
- Existence of an equilibrium in terms of the utility function of the representative agent

### **Cut-through questions**

- The use of martingale and local-martingale representation theorem in perfect replication type results
- Supermartingales and budget constraints and overview
- The use of the support function in the representation of portfolios constraints in pricing and optimization problems