MEK 2500 2015
Week 4: Stress

From the book: **6.5*, 6.7**

E4.1: Assume that the stress tensor in a point $x = (x_1, x_2, x_3)$ is given by

$$\sigma = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 4 & 0 \\ 2 & 0 & 2 \end{pmatrix}$$

1. Find the stress at x on the plane normal to the x_1-axis.
2. Find the stress at x on the plane with normal direction $q = (1, -3, 2)$.
3. Find the principal stresses and principal axes of stress at x.
4. Check that the principal axes of stress are mutually orthogonal.
5. Compute the mechanical pressure of σ.

E4.2*: Take a cantilever beam with a rectangular cross-section occupying the domain $[-a, a] \times [-h, h] \times [0, l]$ with coordinates (x_1, x_2, x_3). Let the end $x_3 = l$ be kept fixed and let the beam be bent by a force f acting in the x_2-direction applied at the free end x_3. Assume that the stress tensor is given by

$$\sigma(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & A + Bx_2^2 \\ 0 & A + Bx_2^2 & Cx_2x_3 \end{pmatrix}$$

1. Determine the relation between A and B if no stress acts on the sides $x_2 = \pm h$.
2. Express the resultant stress on the free end $x_3 = 0$ in terms of A, B and C.
3. Compute the principal stresses and principal axes of stress in terms of A, B, C.
4. Where are the largest principal stresses?

These additional exercises are modified from Continuum Mechanics, A. J. M Spencer, Dover, p.60.