MEK 2500 2015
Week 5: Stress, Part 2

E5.2 and E5.3 given below (thanks to Tormod Landet) will be part of Mandatory Exercise 2. The first exercise below is from Bjørn Gjevik's 2002 lecture notes to the old course MEK2200 and the fourth exercise is from *Continuum Mechanics, A.J.M. Spencer*.

E5.1: Assume that you have a two-dimensional rod occupying the domain $[0, A] \times [-L, 0]$ and that a weight of mass m is evenly attached to the bottom of the rod (where $x_2 = -L$). The weight will act on the rod with force $f = (0, -mg)$ where g is the gravity of earth, and the stress on the bottom surface of the rod (with normal $(0, -1)$) is given by

$$s_n = \frac{f}{A} = (0, -mg/A). \tag{1}$$

Assume that there are no other external forces acting on the rod and that the rod do not experience forces in the x_2 direction. (This is a simplification, but let's go with it for the sake of argument.)

1. What is the normal stress in the plane given by n?
2. What is the shear stress in the plane given by n?
3. What are the stresses in the plane(s) parameterized by $\tilde{n} = (\cos\theta, \sin\theta)$ for $\theta \in [0, 2\pi]$?
4. For what values of θ will the shear stresses be the largest? Give a geometrical interpretation of your finding.

E5.2*: When designing a structure such as a road bridge or a cruise ship out of a ductile material such as steel, one of the most important *dimensioning criteria* in the design is that the structure shall not deform plastically during normal operation. I.e. a road bridge should not be permanently deformed after a large truck has driven over it.

The most common method to help decide if a structure will undergo plastic deformation is to calculate the von Mises stress σ_v from the stress tensor σ_{ij} where the von Mises stress is given as

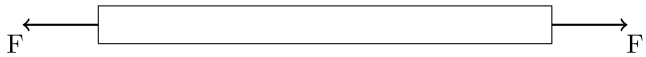
$$\sigma_v = \sqrt{\frac{1}{2}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2)\right]} \tag{2}$$

The von Mises yield criterion is simply $\sigma_v < \sigma_y$ where σ_y is the yield stress of the material. If the von Mises stress is below the material's yield stress then the structure is safe, otherwise it will permanently deform and is considered unsafe. Yield stresses for common materials can be found on page 100 in the textbook.

1. How does the formula for the von Mises stress look when you assume a uniaxial stress situation?

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3}$$

2. Given a uniform rod of metal, a precise ruler and a machine that can apply any force F that you specify to each end of the rod, how would you use this to find the yield stress of the metal in the rod? You are allowed to use the machine multiple times. Assume a uniaxial stress distribution in the rod.



3. Show that the definition of the von Mises stress given by (3) is equivalent to:

$$\sigma_v^2 = \frac{3}{2}\left(\sigma_{\text{dev}} \cdot \sigma_{\text{dev}}\right) \tag{4}$$

where σ_{dev} is the deviatoric stress tensor.

E5.3*: You have designed a huge bridge to cross the Oslo fjord. To prove the safety of your bridge you have calculated the stress tensor everywhere in the bridge. In your calculations you included the weight of the maximum allowed number of cars on the bridge and also the force from the worst hurricane expected in one thousand years at the location.

From your calculation you find that at the most highly loaded location in your bridge the stress tensor is

$$\sigma_{ij} = \begin{bmatrix} 70 & -12 & 0 \\ -12 & -50 & 1 \\ 0 & 1 & 30 \end{bmatrix} \quad \text{[MPa]}. \tag{5}$$

1. Can you build this bridge out of aluminum with the current design?
2. If you want a safety factor of 20% ($\sigma_v < 0.8\sigma_y$) and you decide to build the bridge out of steel, is your design still safe?
3. Is the design optimal, or are you wasting taxpayers money by creating an unnecessarily safe bridge with too much costly steel?

E5.4: Compute the acceleration in a medium of constant density ρ where the stress tensor is given by

$$\sigma_{ij} = \gamma x_i x_j + (\beta - \gamma) \sum_{r=1}^{3} \sum_{s=1}^{3} x_r x_s \delta_{ir} \delta_{js} \tag{6}$$

where δ_{ij} is the Kronecker delta and γ and β are real parameters.