MEK 2500 2015: Mandatory assignment 1

Due date: September 10 2015

1. Please complete the following exercises from the textbook (Physics of Continuous Matter, 2nd edition):

   B.2, B.14,
   C.4,
   7.4, 7.10

2. Imagine a two-dimensional beam occuping the domain $\Omega = [0, L] \times [0, 1] \subset \mathbb{R}^2$ for $L \geq 0$. Denote the coordinates of Ω by $X = (X_1, X_2)$. Assume that the beam is held fixed at one end[1] and that a load is placed on the beam[2] with a strength parameter $k \geq 0$, resulting in a deformation x of the beam of the form

$$x(X) = \begin{pmatrix} x_1(X) \\ x_2(X) \end{pmatrix} = \begin{pmatrix} LkX_1X_2 \\ -kX_1^2 \end{pmatrix} \tag{1}$$

   (a) Compute the displacement field in Lagrangian coordinates defined by:

$$U(X) = x(X) - X \tag{2}$$

   (b) Compute the small-strain strain tensor (in Lagrangian coordinates):

$$\varepsilon(X) = \frac{1}{2}\left(\nabla_X U + \nabla_X U^T\right) \tag{3}$$

   (c) Compute the largest in absolute value principal strain (in Lagrangian coordinates) in the points $A_0 = (L, 1)$ and $A_1 = (0, 1)$.

   (d) Use the provided script to plot the deformation x with $L = 10$ and $k = 0.1$ and the largest principal strain of $\varepsilon(X)$ over the domain Ω.

[1] For instance by attaching it to a wall with superglue
[2] For instance by you standing on the other end