Oppgave 3 (40%)

DEL A

(a) Nevn tre fordeler ved å anvende en sandwichkonstruksjon sammenlignet med en konvensjonell bjelke- eller platekonstruksjon.

(b) Forklar uttrykket *partielle forskyvninger* ("partial deflections") i forbindelse med analyse av en sandwichbjelke.

(c) Hvilke feilmekanismer kan oppstå i en sandwichbjelke eller -søyle?

DEL B

Figur 1 viser en sandwichbjelke AB, med bredde b og lengde L, som er montert som en utkrager. En jevnt fordelt tverrlast q per lengdeenhet er påført bjelken over hele lengden.

Begge skall ("face sheets") kan betraktes som tynne og kjernen som svak (fleksibel).

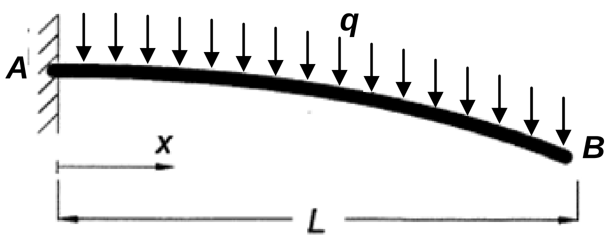
Bjelken har bøyestivhet D og skjærstivhet S. Skjærstivheten S er definert ved

$$S = \frac{T}{\gamma}$$

hvor T er skjærkraft og γ er gjennomsnittlig skjærtøyning over et tverrsnitt.

Ved å bruke partielle forskyvninger, eller annen metode, vis at tverrforskyvningen δ ved den frie enden B er gitt ved

$$\delta = \frac{qL^4}{8D}\left(1 + \frac{4D}{SL^2}\right)$$



Figur 1. Bjelke med tverrlast

DEL C

Figur 2 viser den samme bjelken som i DEL B som nå er påført en aksiallast P ved den frie enden, mens tverrlasten q er fjernet.

(a) Vis at skjærkraften ved innfestningen $T_1 = 0$.

(b) Ved å betrakte den delen av bjelken som ligger mellom et tilfeldig tversnitt og $x = L$, vis at differensialligningen for tverrforskyvningen $w(x)$ er gitt ved

$$\frac{d^2w}{dx^2} + a^2 w = a^2\delta \quad \text{hvor} \quad a^2 = \frac{P}{D}\left(\frac{S}{S-P}\right) \quad \text{og} \quad \delta = w(L)$$

(c) Ved å undersøke ikke-trivielle løsninger for denne ligningen og anvende randbetingelsene ved $x = 0$ og $x = L$, vis at den laveste kritiske verdien av P for knekning av bjelken er gitt ved

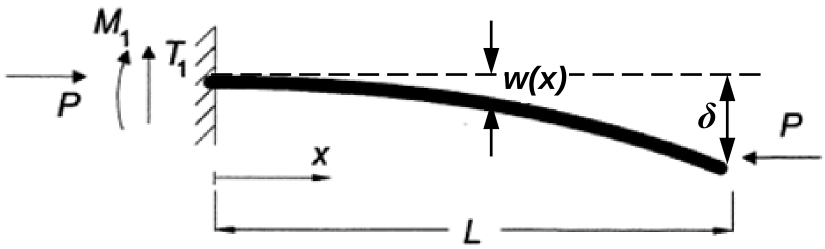
$$P_{kr} = \frac{\pi^2 D}{4L^2 + \pi^2 D/S}$$

Følgende forhold er gitt:

$$w = w_b + w_s\,; \quad -D\frac{d^2 w_b}{dx^2} = M_x\,; \quad T_x = \frac{dM_x}{dx}\,; \quad T_x = S\frac{dw_x}{dx}$$

hvor w_b og w_s er partielle tverrforskyvninger for henholdsvis bøyning og skjær, M_x er bøyemoment og T_x er skjærkraft ved et tilfeldig tversnitt.

Tips for del (c): Siden $T_1 = 0$ er det ingen skjærdeformasjon ved $x = 0$, slik at $\left.\frac{dw_s}{dx}\right|_{x=0} = 0$



Figur 2. Bjelke med aksiallast

MEK 4540 : Oppgave nr. 3

DEL A

(a) • High bending stiffness (for given weight)

• High bending strength (for given weight)

• Smoothe surfaces both sides since no stiffening is needed.

(b) "Partial deflections" is an expression used when deformations due to bending and shear are considered separately and then superimposed to give the total deformation. (The approach is exact for some cases but only approximate for others.)

(c) • Tensile or compressive (or other) failure of faces giving yielding or fracture

• Global buckling, which generally involves a mixture of bending and shear. (Bending buckling and shear crimping are limiting cases when one or the other dominates, i.e. for very slender or stocky columns.)

• Face sheet wrinkling or dimpling

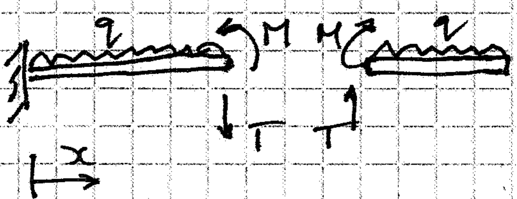
• Local failure due to concentrated loads or at joints

DEL B

The simplest solution is to use partial deflections so

$$w = w_b + w_s$$

where w_b is obtained from the classical bending solution and w_s is the purely shear deformation.



By equilibrium of right-hand part,

$$M = q\frac{(L-x)^2}{2}, \quad T = q(L-x)$$

Bending: $$-D\frac{d^2w_b}{dx^2} = M = -q\frac{(L-x)^2}{2} \quad \text{i.e.} \quad D\frac{d^2w_b}{dx^2} = q\tfrac{1}{2}(L-x)^2$$

Integrate: $$D\frac{dw_b}{dx} = -\frac{1}{6}q(L-x)^3 + A$$

Again: $$D\,w_b = \frac{1}{24}q(L-x)^4 + Ax + B$$

Boundary conditions:

$$\frac{dw_b}{dx} = 0 \text{ at } x = 0 \rightarrow A = \frac{1}{6} q L^3$$

$$w_b = 0 \text{ at } x = 0 \rightarrow 0 = \frac{1}{24} q L^4 + \cancel{\frac{1}{6} q L^3 \times 0} + B$$

$$B = -\frac{1}{24} q L^4$$

Deflection at $x = L$:

$$\delta_b = w_b(L) = \frac{1}{D} \left\{ AL + B \right\} = \frac{1}{D} \left(\frac{1}{6} q L^4 - \frac{1}{24} q L^4 \right)$$

$$= \frac{1}{D} \cdot \frac{q L^4}{8}$$

Shear: $\gamma = \frac{T}{S} = \frac{q}{S} (L - x)$

$$w_s = \int_0^x \gamma \, dx$$

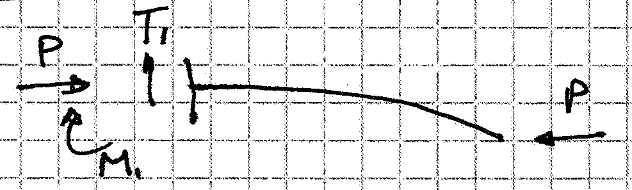
$$\delta_s = w_s(L) = \int_0^L \frac{q}{S} (L - x) \, dx = \frac{q}{S} \left[Lx - \frac{1}{2} x^2 \right]_0^L$$

$$= \frac{q}{S} \cdot \frac{1}{2} L^2$$

Total deflection $\delta = \delta_b + \delta_s = \underline{\underline{\frac{q L^4}{8D} \left(1 + \frac{4D}{S L^2} \right)}}$

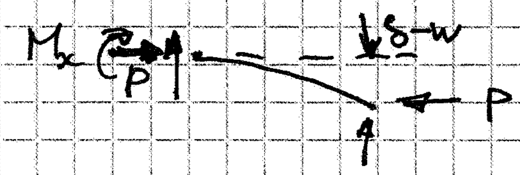
DEL C

(a)



Vertical equilibrium of forces on entire beam: T_1 is the only force so $T_1 = 0$

(b)



Moment equilibrium of part to right of cut: Take moments about cut.

$$M_x + P(\delta - w) = 0$$

$$M_x = Pw - P\delta$$

We have $\quad w = w_b + w_s$

$$\text{so} \quad \frac{d^2 w}{dx^2} = \frac{d^2 w_b}{dx^2} + \frac{d^2 w_s}{dx^2}$$

But $\quad -D\dfrac{d^2 w_b}{dx^2} = M_x \quad$ (bending equation)

and $\quad \dfrac{dw_s}{dx} = \dfrac{1}{S} T_x \quad$ (shear equation)

$$\text{so} \quad \frac{d^2 w_s}{dx^2} = \frac{1}{S}\frac{dT_x}{dx} = \frac{1}{S}\frac{d^2 M_x}{dx^2} \quad \left(\text{since } T_x = \frac{dM_x}{dx}\right)$$

$$\text{Hence} \quad \frac{d^2 w}{dx^2} = -\frac{M_x}{D} + \frac{1}{S}\frac{d^2 M_x}{dx^2}$$

$$= \frac{P\delta - Pw}{D} + \frac{1}{S} P \frac{d^2 w}{dx^2}$$

Rearrange: $\quad \left(1 - \dfrac{P}{S}\right)\dfrac{d^2 w}{dx^2} + \dfrac{P}{D} w = \dfrac{P}{D}\delta$

$$\text{i.e.} \quad \frac{d^2 w}{dx^2} + \frac{P}{D}\left(\frac{S}{S-P}\right) w = \frac{P}{D}\left(\frac{S}{S-P}\right)\delta$$

With $\quad a^2 = \dfrac{P}{D}\left(\dfrac{S}{S-P}\right) \quad$ this becomes

$$\frac{d^2 w}{dx^2} + a^2 w = a^2 \delta$$

Solution is

$$w = C_1 \sin ax + C_2 \cos ax + \delta$$

Boundary conditions are $\frac{dw_b}{dx} = 0$ at $x = 0$

$w = 0$ at $x = 0$

But $\frac{dw_s}{dx} = 0$ at $x = 0$ (see footnote to question) so first condition is $\frac{dw}{dx} = 0$ at $x = 0$.

This gives $C_1 a \cos(0) + -C_2 \sin(0) = 0$

i.e. $C_1 = 0$

$w = 0$ at $x = 0$ gives $0 = C_2 + \delta$, i.e. $C_2 = -\delta$

We note also that $w(L) = \delta$, so

$$\delta = \cancel{C_1 \sin aL} + C_2 \cos aL + \delta$$

This gives $C_2 \cos aL = 0$

so $C_2 = 0$ (trivial solution) or $\cos aL = 0$

Non-trivial solution possible only if $aL = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

i.e. $aL = \frac{2n-1}{2}\pi \quad n = 1, 2, 3, \ldots$

But $a^2 = \frac{P}{D}\left(\frac{S}{S-P}\right)$ so P_{cr} given by

$$(S-P) D a^2 = PS \quad \text{i.e.} \quad P(S + Da^2) = SDa^2$$

$$P_{cr} = \frac{SDa^2}{S + Da^2} = \frac{\left(\frac{2n-1}{2}\right)^2 \pi^2 SD/L^2}{S + \left(\frac{2n-1}{2}\right)^2 \pi^2 D/L^2}$$

Lowest given by $n = 1$

so $$P_{cr} = \frac{\frac{1}{4}\pi^2 SD/L^2}{S + \frac{1}{4}\pi^2 D/L^2} = \frac{\pi^2 D}{4L^2 + \pi^2 D/S}$$