COMPULSORY EXERCISE IN STK4510, AUTUMN SEMESTER 2013

MUST BE HANDED IN BEFORE THURSDAY 31.10.2013 AT 14.30

Students who want to do the exam in STK4510 must pass the compulsory exercise. In order to pass, *all* questions must be answered satisfactory. You can use the software package that you prefer for solving the exercises (R, Matlab, Excel...) Deadline for handing in your compulsory exercise is Thursday 31.10.2013 at 14.30, at the office of the Mathematics Institute, Nils Henrik Abels hus, 7th floor.

Task 1. In this exercise you are supposed to estimate the two parameters *expected return* μ and *volatility* σ in a geometric Brownian motion

$$S_t = S_0 \exp(\mu t + \sigma B_t)$$

Here, B is a Brownian motion.

a) Download a data set of daily closing stock prices of your "favourite" company. For this purpose, you can for example use the link http://finance.yahoo.com/ to get historical price series. Describe your data set, and plot the prices as a time series. Compute the logreturn data, and plot these as well. Estimate μ and σ based on logreturns, and report these estimates. What are the annualized expected return and volatility?
b) Compute the *returns*, and estimate the mean and standard deviation of this time series. Compare your results with the estimates for the logreturns, and comment on what you find.
c) Plot the autocorrelation function for the logreturns and the squared logreturns. Are the logreturns independent? Why/why not?

Task 2. In this exercise you are supposed to create your own option price calculator.

a) Implement, based on the Black & Scholes formula, a calculator that takes as input the volatility, the risk-free interest rate, the strike, the current price of the underlying and the time to exercise, and computes the call option price.
b) Report the option prices for calls on the OBX index for a given day, ranging over the different strikes and exercise times (you find prices on Oslo Børs homepage. If there is no trading price, use the average of the "Buyer" and "Seller" prices). Use your calculator to find the *implied volatility*, that is, the value of σ that you must use in order to get the same price as the one reported in the market. Give reasons for your choice of the risk-free interest rate. Report your implied volatilities graphically as a function of time to exercise and strike.

Task 3. In this exercise you are going to investigate the price of an Asian-style call option written on the stock that you analysed in Exercise 1.

Consider an Asian call option with averaging period from $T_1 = 20$ to $T_2 = 30$ days, strike $K = 100$ and exercise time $T = T_2$. Hence, the payoff from this option is

$$\max\left(\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S_t\, dt - K, 0\right)$$

There is no "analytic" pricing formula for such options, and one must either simulate the price or use approximation methods. You will analyse both.

a) First, compute the price using Monte Carlo simulation. You should first read about Monte Carlo simulation of option prices (in particular Asian options) in Chapter 5 of Benth. The price is found by simulating the expectation

$$P = e^{-rT}\mathbb{E}_Q\left[\max\left(\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S_t\, dt - K, 0\right)\right]$$

where, with respect to the probability Q,

$$dS_t = rS_t\, dt + \sigma S_t\, dW_t \tag{1}$$

In the exercise, you need to argue for which interest rate r you should use. To make matters simple, please let $S_0 = 100$ (although it probably was not 100 in the data series you studied in exercise 1!). How reliable is you price, depending on the number of simulations?

b) As you probably experienced in task a), it requires a lot of computational work to simulate the price. A much simpler approach is to use a variation over the Black-Scholes formula! This goes as follows: Let X be a normally distributed random variable with mean a and variance b^2. Find a and b such that

$$\mathbb{E}[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S_t\, dt] = \mathbb{E}[\exp(X)]$$

and

$$\mathbb{E}[(\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S_t\, dt)^2] = \mathbb{E}[\exp(2X)]$$

In these calculations, use S_t as given in (1). This is called *moment-matching*, as we find a lognormal random variable $\exp(X)$ which matches the first two moments of the average stock price. A hint for the computation of the second moment is to observe that

$$(\int_{T_1}^{T_2} S_t\, dt)^2 = \int_{T_1}^{T_2}\int_{T_1}^{T_2} S_t S_u\, dt\, du$$

For X with given a and b, you can now do the following approximation:

$$\begin{aligned} P &= e^{-rT}\mathbb{E}_Q\left[\max\left(\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S_t\, dt - K, 0\right)\right] \\ &\approx e^{-rT}\mathbb{E}[\max(\exp(X) - K, 0)] \end{aligned}$$

Compute a formula a-la Black & Scholes for the last expression, and insert the values you have found to find a price. How is this comparing with the price you simulated in task a)?