Introduction and Techniques
in Financial Mathematics
UiO-STK4510
Autumn 2015
Teacher: *S. Ortiz-Latorre*

Compulsory Exercise

Students who want to do the exam in STK4510 must pass the compulsory exercise. In order to pass, **all** questions must be answered satisfactory. You can use the software package that you prefer for solving the exercises (R, Matlab, Excel...)

Deadline: *November 17, 2015 at 14:00.* To be handed in at the administration office of the Department of Mathematics, University of Oslo, 7th floor, Niels Henrik Abel building.

Task 1

Download a series of daily stock prices of your favorite company or any stock index (the series should span at least two years). You may find such series at "yahoo finance", for example. In this task you will check the stylized facts of financial series and fit your data to a geometric Brownian motion.

a) Plot the series of prices and logreturns.

b) Fit the data to a geometric Brownian motion estimating the mean μ and the volatility σ. Compute also the skewness and kurtosis. Give μ and σ assuming that we measure the time in years.

c) Check for normality of logreturns: Plot a QQ-plot, plot a histogram together with a kernel density estimation and a normal density with the parameters estimated in the previous section. Finally, use the Jarque-Bera test. Discuss.

d) Plot the empirical autocorrelation function of the logreturns and the empirical autocorrelation function of the squared logreturns. Use the Ljung-Box test to check the independence of logreturns. Discuss.

e) Repeat the previous analysis for monthly logreturns and compare the results.

1 Task 2

In this task you will do some computations on one of the first models for stochastic interest rates. Consider the process $\{r_t\}_{t\in\mathbb{R}_+}$ given by the stochastic differential equation (s.d.e)

$$dr_t = k(\theta - r_t)dt + \sigma dW_t, \quad r_0 \in \mathbb{R}_+, \tag{1}$$

where k, θ and σ are strictly positive constants and W is a standard Brownian motion.

a) First find the solution $\{Z_t\}_{t\in\mathbb{R}_+}$ of the s.d.e.

$$dZ_t = -aZ_t dt + \sigma dW_t, \quad Z_0 \in \mathbb{R},$$

as a function to the initial condition Z_0, where a and σ are strictly positive constants.

b) For any $\beta \in \mathbb{R}$ define the process $Y_t = Z_t + \beta$. Find the s.d.e. satisfied by Y. Choosing appropriately the parameters $a \in \mathbb{R}_+$ and $\beta \in \mathbb{R}$, find the solution $\{r_t\}_{t\in\mathbb{R}_+}$ of the s.d.e. (1).

c) Compute $\mathbb{E}[r_t]$ and $\mathrm{Var}[r_t]$. What is the law of r_t?

d) r_t takes negative values with strict probability and, therefore, it is not a good model for interest rates. A possible solution is to consider $R_t = \exp(r_t)$. Compute the dynamics of R_t, i.e., find the s.d.e. describing the evolution of R_t

2 Task 3

In this task you will work on the relationship between a call option and a put option on the same asset, with the same strike and with the same exercise time. Recall that a call option is a contingent claim H_1 with $H_1 = h_1(S_T) = \max(0, S_T - K)$. A put option is a contingent claim H_2 with $H_2 = h_2(S_T) = \max(0, K - S_T)$.

a) Let C_t and P_t denote the price at time t of a call option and a put option with the same strike price K and exercise time T. Assume that those contingent claims are traded in the market in addition to the riskless asset and risky asset S_t. Assuming only that there are no arbitrage oportunities in the market (we do not specify any particular dynamics for S), deduce that

$$C_t - P_t = S_t - e^{-r(T-t)}K, \quad 0 \le t \le T. \tag{2}$$

b) Assume now that we are in the Black-Scholes model. Use the Black-Scholes formula for pricing contingent claims of the form $h(S_T)$ to compute the price of a put option P_t and its hedging strategy. Compute also the greeks.

c) Check that formula (2) holds in the Black-Scholes model.