Measuring inequality - Week 8

ECON1910 - Poverty and distribution in developing countries

Readings: Ray chapter 6

22. March 2011

Why care about economic inequality?

- Ethical grounds
  Inequalities often start the day the children are born regardless of their choices.

- Functional reasons
  Inequality has an impact on other economic features that we care about.

Inequality in what?

- Income flows?
- Wealth/asset stocks
- Lifetime income

Measuring inequality

- How do we measure inequality?
- How do we rank alternative distributions with respect to how much inequality they embody?

Criteria for inequality measurement

- Which is more unequal division of the cake between 3 persons:
  22 - 22 - 56? or
  20 - 30 - 50?

- Denote:
  $n =$ the number of individuals in the economy
  $y_i =$ income received by individual i
  Income distribution - $(y_1, y_2, ..., y_n)$
  Inequality $I\ (y_1, y_2, ..., y_n)$

Four criteria for inequality measurement

1. Anonymity principle
   - It does not matter who is earning the income.
2. Population principle
   - Population size does not matter, only the proportions of the population that earn different levels of income.
3. Relative income principle
   - Only relative income should matter, not the absolute ones.
4. Dalton principle
   - If one income distribution can be achieved from another by constructing a sequence of regressive transfers, then the former distribution must be deemed more unequal than the latter.

The Dalton Principle

- A regressive transfer is any transfer of income from the "not richer" individual to the "not poorer" individual.

- If one income distribution can be achieved from another by a process of regressive transfers, then the former distribution must be more unequal than the latter.

- The Dalton Principle:
  For every income distribution and every (regressive) transfer $d > 0$, $I(y_1, y_2..., y_i, ...y_j..., y_{n)} < I(y_1, y_2..., y_i - d, ...y_j + d, ..., y_{n)}$ for $y_i \leq y_j$

Two Dalton Explanations

1. If some distribution outcome A can be achieved from a distribution outcome B by a regressive transfers, then A must be more unequal than B.
   - If B$\longrightarrow$A by regressive transfers, then for an inequality index $I\ ()$
   - $I\ (B) < I\ (A)$
2. A regressive transfer will result in higher inequality: if there is a distribution outcome B subjected to a regressive transfer, the inequality of B would rise.
   - Denote regressive transformation as $\hat{B}$
   - Then $B \rightarrow \hat{B} \Longrightarrow I(B) < I(\hat{B})$

Questions.

- Why not say ‘poorer’ or ‘richer’ instead of ‘not richer’ and ‘not poorer’
- Define y_i as "poorer" and y_j as "richer" then, $y_i < y_j$.
- The Dalton Principle does indeed work for cases of equality.
- Assume that an economy of perfect equality exists. One regressive transfer makes the economy more unequal.
- Can we define the Dalton Principle using progressive transfers instead?"
- No

Two Counter Examples With Numbers.

- Suppose a perfectly equal economy of five individuals, earning (20; 20; 20; 20; 20)
- Using a progressive transfer (as defined as taking from not poorer and giving to not richer), one person loses a dollar, and another gains a dollar: (21; 20; 20; 20; 19).
- Inequality goes up!
- Suppose an economy of five individuals exists such that income is (19; 20; 20; 20; 21). Let a progressive transfer that take $3 from the richest individual and give $3 to the poorest individual: (22; 20; 20; 20; 18).
- Can a regressive transfer ever decrease inequality? No!

Four criteria for inequality measurement

1. Anonymity principle
   - The function I is completely insensitive to all permutations of the income distribution $(y_1, y_2, ..., y_n)$ among the individuals
2. Population principle
   - $I(y_1, y_2, y_3 ..., y_n) = I(y_1, y_2, y_3 ..., y_n; y_1, y_2, y_3 ..., y_n)$
3. Relative income principle
   - $I(y_1, y_2, y_3 ..., y_{n)} = I(\delta y_1, \delta y_2, \delta y_3 ..., \delta y_n)$
   - Income shares are all we need: poorest x% earn y%.
4. Dalton principle
   - For every income distribution and every (regressive) transfer $d > 0$, $I(y_1, y_2 ..., y_i, ... y_j ..., y_{n)} < I(y_1, y_2 ..., y_i - d, ... y_j + d, ..., y_{n)}$ for $y_i \leq y_j$

Income distribution by population and income shares

- Look at the following income distribution:

Individual	Adam	John	Emily	Mark	Ted	Total
Income	400	600	1300	2700	5000	10000
Quintile	1st	2nd	3rd	3th	5th	All
% of income	4	6	13	27	50	100

- If we double the income to everyone, the distribution does not change:

Individual	Adam	John	Emily	Mark	Ted	Total
Income	800	1200	2600	5400	10000	20000
Quintile	1st	2nd	3rd	3th	5th	All
% of income	4	6	13	27	50	100

Income distribution by population and income shares

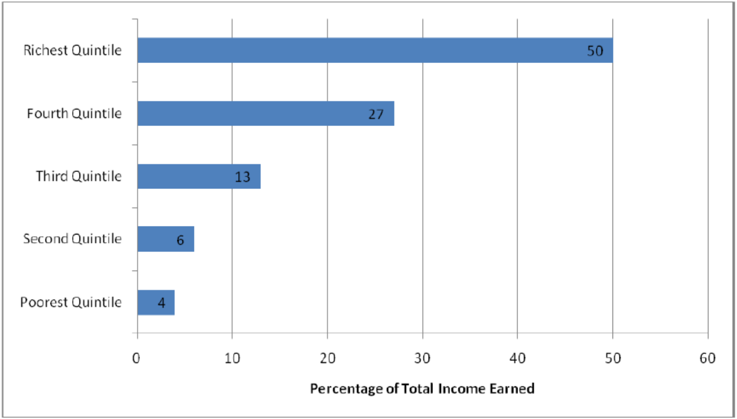
- Let us verify that this income distribution is the same as the last one:

individual	1	2	3	4	5	6	7	8	9	10	Total
Income	400	400	600	600	1300	1300	2700	2700	5000	5000	2000
% of income	2	2	3	3	6.5	6.5	13.5	13.5	25	25	100

- From this table we can write the same information in quintiles:

Quintile	1st	2nd	3rd	3th	5th	All
% of income	4	6	13	27	50	100

Income distribution by population and income shares



This figure represent the same income distribution as in all of the tables presented above

Cumulative Population and Cumulative Income

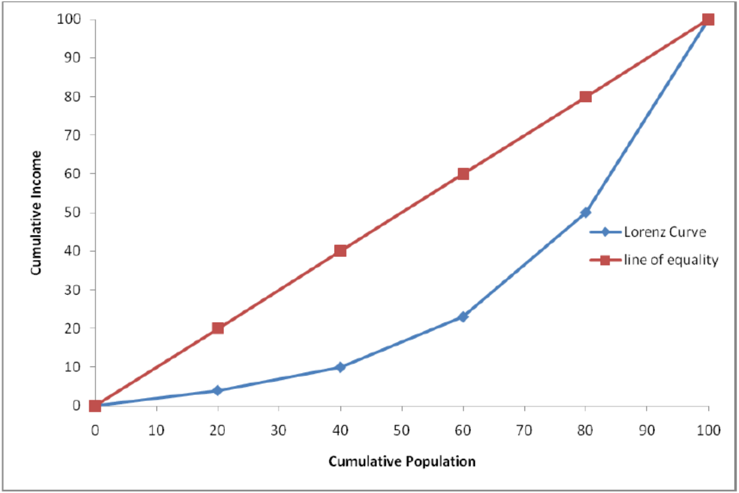
Cumulative Population	0%	20%	40%	60%	80%	100%
Cumulative Income	0%	4%	10%	23%	50%	100%

This table contains the same information as all the tables presented above

The Lorenz Curve

- Common graphical method of illustrating the degree of income inequality in a country.
- Shows the relationship between the percentage of income recipients and the percentage of income they receive.

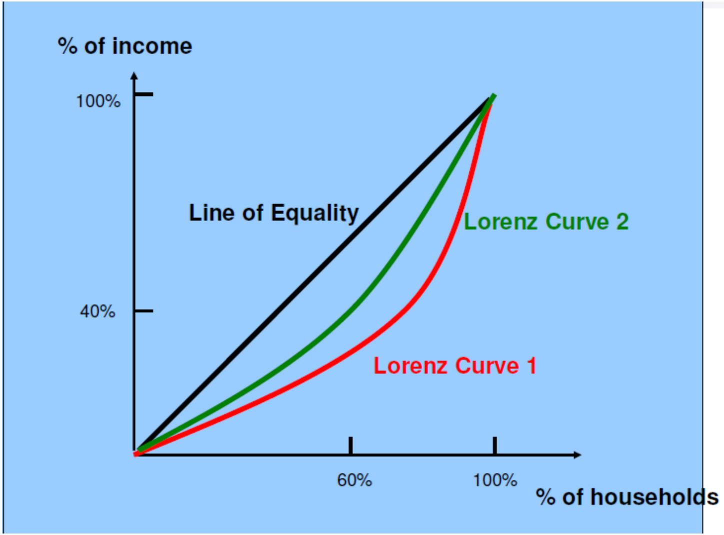
The Lorenz Curve



Lorenz criterion

- An inequality measure I is Lorenz-consistent if for every pair of income distributions (of y's and z's)
  $I(y_1, y_2, y_3..., y_{n)} \gtrless I(z_1, z_2, z_3..., z_n)$
  the Lorenz curve of $(y_1, y_2, y_3..., y_n)$ lies everywhere to the right of $(z_1, z_2, z_3..., z_n)$
- An inequality measure is consistent with the Lorenz criteria if and only if the 4 criteria above are simultaneously holding.

The Lorenz Curve

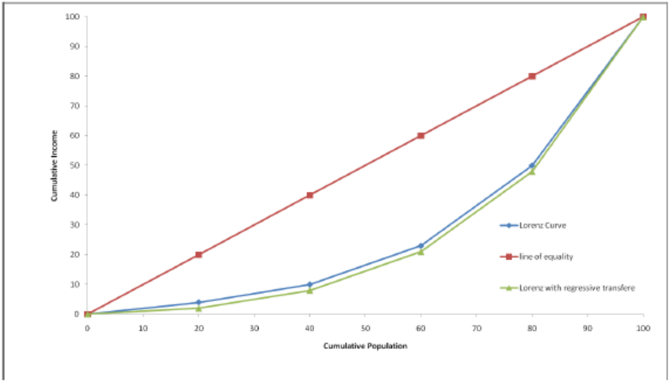


A regressive transfer

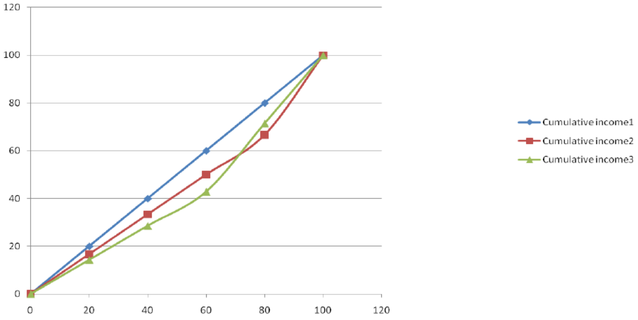
Individual	Adam	John	Emily	Mark	Ted
Income	200	600	1300	2700	5200
Quintile	1st	2nd	3rd	3th	5th
% of income	2	6	13	27	52

Cumulative population	20	40	60	80	100
Cumulative Income	2%	8%	21%	48%	100%

The Lorenz curve with a regressive transfer



Lorenz crossing



The Lorenz criteria does not apply.

Lorenz crossing

- If Lorenz curves are crossing:
  - The Dalton principle does not apply
  - There must be both "progressive" and "regressive" transfers in going from one distribution to the other.

Complete measures of inequality

- Two problems with the Lorenz curves
  1. Policy makers and researchers are often interested in summarizing inequality by a number.
  2. When Lorenz curves cross, they cannot provide useful inequality ranking.

- An inequality measure that spits out a number for every conceivable income distribution can be thought of as a complete ranking of income distributions

- The different "complete measures" might disagree in ranking

Complete measures of inequality - Notation

- m - distinct incomes
- j - a specific income class
- n_j - number of individuals in income class j
- $\sum_{j=1}^{m}$ - the sum over the income classes 1 through m
- $\sum_{j=1}^{m} n_j$ - The total number of people n
- μ - the mean of any income distribution
- $\mu = \frac{1}{n} \sum_{j=1}^{m} n_j$

1. The range

The difference in incomes of the richest and the poorest individuals, divided by the mean

$$R = \frac{1}{\mu}\left(y_m - y_1\right)$$

- Ignore all income between the richest and the poorest
- Can be insensitive to the Dalton principle.

2. The Kuznets ratios

- The ratio of the shares of incomes of the richest x% to the poorest y%, where x and y stand for numbers such as 10, 20 or 40
- Does not consider the whole income distribution
- Can be insensitive to the Dalton principle

3 The mean absolute deviation

- Takes advantage of the entire income distribution
- Inequality is proportional to distance from the mean income
- Take all income distances from the average income, add them up, and divide by total income

$$M = \frac{1}{n\mu} \sum_{j=1}^{m} n_j \left| y_j - \mu \right|$$

- Often insensitive to the Dalton principle.

4. The coefficient of variation

- Gives more weight to larger deviations from the mean than the "Mean absolute deviation"

$$C = \frac{1}{\mu}\sqrt{\sum_{j=1}^{m}\frac{n_j}{n}\left(y_j - \mu\right)^2}$$

- It satisfies all four principles and is therefore Lorenz-consistent.

5. The Gini coefficient

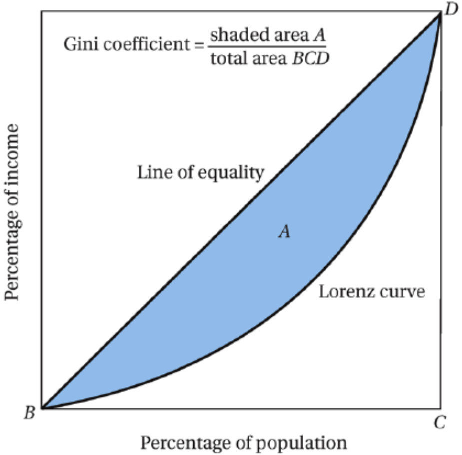
- Widely used in empirical work.
- Takes the difference between all pairs of incomes and simply totals the (absolute) difference.
- It is as if inequality is the sum of all pairwise comparisons of "two-person inequalities".
- The Gini coefficient is normalized by dividing by population squared (because all pairs are added and there are n^2 such pairs) as well as mean income

$$G = \frac{1}{2n^2\mu} \sum_{j=1}^{m} \sum_{k=1}^{m} n_j n_k \left| y_j - y_k \right|$$

- It satisfies all four principles and is therefore Lorenz-consistent.

The Gini coefficient

The Gini coefficient is precisely the ratio of the area between the Lorenz curve and the line of perfect equality, to the area of the triangle below the line of perfect equality.



Why both Gini and Coefficient of variation

- They are both Lorenz-consistent
- If Lorenz curves does not cross - they will always give the same ranking
- If Lorenz curves cross - they might give different ranking