Short solution proposal to the compulsory assignment in ECON1910

Problem 1: Harrod-Domar vs. Solow. In the Harrod-Domar model a change in the savings rate \( s \) has a permanent effect on the growth rate of GDP per capita, while in the Solow model a change in the savings rate has only a temporary effect on the growth rate of GDP per capita. Why is this the case?

**Answer:** The main difference between the Harrod-Domar (HD) model and the Solow model is that HD assumes constant marginal returns to capital, while Solow assumes decreasing marginal returns to capital. The reason that a change in the savings rate has a permanent effect in HD, while only a temporary effect in Solow, is exactly due to the differences in assumptions on the marginal returns to capital.

To see why, assume that we initially are in the steady state in the Solow model, where investments exactly are equal to break-even investments (i.e. the amount of investments \( sy_t \) are equal to \( (n + \delta)k_t \), the amount of investment that needs to be undertaken in order for the capital stock per capita next period to be the same size as today). If we increase the savings rate in the Solow model from \( s \) to \( s' \), we will in the next period have more capital per capita than before, as depreciation (\( \delta \)), population growth (\( n \)) and capital today (\( k_t \)) are the same, i.e. break-even investments today do not change. This additional capital will generate more output next period (a fraction \( s' \) of which is saved), but we will also need to invest more next period if we were to keep capital constant at this new level since the new break-even investment level \( (n + \delta)k_{t+1} \) > \( (n + \delta)k_t \) since \( k_{t+1} > k_t \).

It will now (in period \( t + 1 \)) also be the case that investments are higher than the new break-even investment level, but less so than last period because the marginal product of capital is lower at the new and higher level of \( k \). As the marginal product of capital decreases as \( k \) gets larger, while the ‘cost’ of higher \( k \) in terms of higher break-even investments increases linearly with \( k \), the temporary effect on growth of the change in \( s \) will gradually level off until we reach the new steady state, where growth again is 0.

Note that the last argument does not hold for the HD model. In the HD model the marginal product of capital per capita is constant, and hence a permanent change in \( s \) will have a permanent effect on the growth rate of the economy.

Problem 2: Population growth rates in the Solow model. Define and explain “steady state” in the Solow model. Assume that the economy initially is in the steady state. Analyze the short-run and long-run effects of a change in the population growth rate (\( n \)) on per capita GDP growth rates and levels in the Solow model, everything else equal.

**Answer:** Steady state: The state the economy eventually converges to for arbitrary initial conditions, identified by constant growth rates (here 0).

The easiest way to answer this question is to do a graphical analysis of the Solow model, see Figure 1 and the slides accompanying Lecture 4 (Note that on the exam you will have to define all the variables you are using and explain what the different curves in the figure represent. We do not do that here, however, because all the definitions and the explanation of the figure is in the slides accompanying Lecture
Figure 1: The effect of a change in population growth from \( n \) to \( n' \) in the Solow model

4). In Figure 1 we assume that we initially are in the steady state \((k^*, y^*)\). Then the population growth rate changes from \( n \) to \( n' \). We see that this changes the slope of the line denoting break-even investments—it becomes steeper. The new intersection between the break-even investment curve and the savings curve (the steady state) is now at the point \((k'^*, y'^*)\), and we see that \( k'^* < k^* \) and \( y'^* < y^* \). The long term effect of a change in population growth from \( n \) to \( n' \), where \( n' > n \), is hence that GDP per capita falls from \( y^* \) to \( y'^* \). We have thus found that a change in the population growth rate has a level effect, the steady state level of \( y \) changes as \( n \) changes. The growth of \( y \) in steady state does not change, however. Population growth hence has no growth effect (in steady state). In the transition from the old to the new steady state, there will be negative growth in \( y \). (Note that on the exam this explanation will not be enough. You also need to provide the intuition for the results, see next paragraph.)

Now why is this the case, why does the steady state level of \( y \) become lower as the population growth increases? To maintain the same level of capital per capita next period, the current generation has to invest more than before (the ‘cake’ has to be larger if everybody is to have the same amount of cake as before). This will only be possible if the savings rate \( s \) changed, which it by assumption does not do here. This means that the amount of capital per capita has to go down over time, as the the amount of investment is less that what the society need to invest in order to keep the capital stock constant. But the key here is that as the capital stock becomes lower and lower, the marginal return of the capital increases (due to the assumption about decreasing marginal return to capital), and this is what ensures that this process of capital de-accumulation will converge and hit the new steady state.

**Problem 3: Data and PPP.** To answer this question, you will need to download data from the World Bank’s DataBank.¹

(a) Download the data series GDP per capita (constant 2000 US$) and GDP per capita,

Figure 2: GDP per capita in China and Norway in PPP terms and exchange rate terms.

PPP (constant 2005 international $) for China and Norway for the years 1980–2010.\(^2\)
Plot PPP GDP per capita against time for China and Norway in one graph and GDP per capita in constant 2000 US$ for China and Norway in another graph.\(^3\) Discuss.

**Answer:** See Figure 2. Here you should notice that the gap between the lines is smaller in Figure 2(a) than in Figure 2(b)—this is the effect of PPP vs. exchange rate denomination of GDP per capita (the explanation for this is given in problem 3(c)). Notice, however, that you cannot compare the level of GDP per capita across the two figures, as Figure 2(a) is denoted in 2005 USD and Figure 2(b) is denoted in 2000 USD, i.e. the value of a dollar (in the US) is not the same in both figures.

(b) Define purchasing power parity (PPP). If we want to compare the standards of living in China and Norway, should we compare GDP in PPP terms or in US$ terms?

**Answer:** The PPP approach is to create an artificial exchange rate that ensures purchasing power parity, i.e. an exchange rate that is such that one dollar can buy the same amount of goods in all countries. If we want to compare living standards for the general population we should therefore use PPP terms, not exchange rate terms, but if we (for some reason) were interested in the consumption of the rich it could potentially be more relevant to use the exchange rate, as the rich often consume more traded goods (wine from France, cars from the U.S. etc) than others.

(c) In 2010, GDP per capita in current US$ was 4,428 in China and 84,538 in Norway. The same figures using current PPP$ were 7,598 and 56,691.\(^4\) Why is Norway’s GDP per capita higher in current US$ than in PPP$, while the opposite is true for China?

\(^2\)Note that constant 2000 US$ means that the World Bank has converted the GDP figure for each year at that year’s market exchange rate, and thereafter normalized these GDP figures such that US inflation has been removed. The same thing holds for PPP (constant 2005 international $), but here the World Bank have used the PPP exchange rate, not the market exchange rate.

\(^3\)It is probably easiest to do this in Microsoft Excel, but you can use any program you want.

\(^4\)Note that these figures are not the same as the ones you downloaded from the World Bank’s DataBank.
Answer: China’s GDP is higher in PPP terms than when measured using the exchange rate, while for Norway the opposite is true. One of the main reasons is the following (the Balassa-Samuelson effect): The production of the economy can be divided into two sectors: the traded sector (the sectors producing goods that are traded internationally) and the non-traded sector (the sectors producing goods that are only traded domestically). The productivity differences (per working hour, for example) across countries in the traded sector (manufacturing and the like) are much larger than the productivity differences across countries in the non-traded sector (services and the like). As the exchange rate is a price that is determined by the flow of goods (and capital) across borders, the exchange rate will hence only reflect the differences in productivity in the traded sector. The value of the production of non-traded goods will therefore be ‘inflated’ in countries that have a high productivity in the traded sector if we use the exchange rate to determine the value of these non-traded goods, and the opposite hold for the countries with relatively low productivity in the traded sector. Since the PPP approach is to create an artificial exchange rate such that the purchasing power of a dollar is the same across countries (measured using a basket of goods that consist of both traded and non-traded goods), the GDP in PPP terms will be lower than GDP using the exchange rate for countries that have a very productive traded sector, and opposite apply for those countries that have a relatively less productive traded sector.

Problem 4: Migration. Discuss the suitability of the models analyzed in Problem 1 to explain economic growth in cities versus in agricultural development in rural areas. Explain the consequences if economic growth is strong in urban areas but weak in rural areas.

Answer: The standard models of growth studied in Problem 1 explain growth by the accumulation of (physical) capital. This is probably most directly suited to explain growth through industrialization, i.e. the construction of factories and other physical means of production. For this reason, we may say that standard growth models are best suited at explaining urban development. Growth in rural areas is mostly through improved agricultural production. To some extent, this may be explained by increased capital stock, e.g. the availability of tractors. However, agricultural development is more dependent on the adoption of better yielding seeds, use of fertilizer, etc. which is less clearly connected with investments in physical capital. The use of these could be seen as a product of improved human capital (more knowledge), so to understand rural development it may be necessary to extend the models to wider definitions of capital. Finally, it could be mentioned that rural-urban migration may assure that there is always available labor as assumed in the Harrod-Domar model.

If growth is stronger in the cities than in rural areas, this implies that we should have migration into the cities, and this would improve the conditions of both the migrants and those remaining in the rural areas. However, if there is imprecise knowledge about conditions in the cities, or wages in the cities are inflexible and stuck on a high level as in the Harris-Todaro model we may get more migrants than there are urban jobs, and hence the creation of an urban informal sector. As the productivity in this sector is typically low and living standards bad, this is a problem both from an economic and a social point of view.

The reason is that these are denoted in 2010 US$, while in (a) and (b) we used constant 2000 and 2005 prices for PPP$ and US$. 
Problem 5: Inequality and growth. Explain how the distribution of assets (capital) affect the growth process in the models analyzed in Problem 1. Discuss how realistic you think this is, and present some potential modifications of the models that may alter the conclusion. Finally, discuss critically how inequality affect growth empirically.

Answer: In standard growth models, the distribution of assets (i.e. capital) is not explicitly modeled, instead it is more or less implicitly assumed that capital is distributed efficiently among workers. If capital markets are well functioning, one can show that this would be achieved through lending and borrowing as well, hence the assumption is less extreme than it may seem.

However, it is far from clear that capital markets are perfect, particularly in poor countries. If this is not the case, the mechanisms by which the rich can let the poor use their capital is removed and distribution will matter. A quite standard assumption is that a worker who has little capital has a high return to additional capital, whereas a worker with much capital has lower returns. Hence it is profitable for the worker with the largest amount of capital to lend some of his to the poorer worker in the sense that the increase production is high enough to make both better off. To introduce this in the model, we can simply have a distribution of capital and no credit markets. One can then show that production at any point in time will depend on the distribution of capital, the more unequal it is, the lower would total production be. Other channels through which distribution may affect growth is through effects on saving rates, political outcomes (more populist and hence less efficient policies with high inequality), and effects through the labor market.

The simplest way to study this empirically is to look at the distribution of income (or wealth, but data are difficult to find) in say 1970, and look at subsequent economic growth (say 1970-2005). If we do this, we find that unequal countries grow more slowly than more equal countries, controlling for other factors that also affect growth. A problem with this approach is that there may be factors that are left out of the analysis (social coherence, culture, etc.), and the results are also mainly driven by a comparison of Latin America vs. South-East Asia. To fix this, we can try to follow countries over time. Then the results seem to be the opposite, that inequality is good for growth. However, there are also results indicating that mere changes in inequality (both up and down) reduce subsequent growth. Hence we do not have a final conclusion.