Measurement of poverty and inequality

Econ 1910 – Poverty and distribution in developing countries

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Introduction

- How many are poor?
- How large is inequality?
- How did the financial crises affect poverty?

To answer these questions, we need summary measures.

- to understand the present
- to compare over time
- to compare across countries

Introduction

In order to make useful summary measures, we need to specify what we want them to capture.

- How should a poverty measure respond to changes in income
  - among the poor?
  - among the rich?
  - among the middle class?

Introduction

In order to make useful summary measures, we need to specify what we want them to capture.

- How should a poverty measure respond to changes in income
  - among the poor?
  - among the rich?
  - among the middle class?
- How should an inequality measure respond to these changes?

Introduction

In order to understand statements like *"poverty has decreased"*

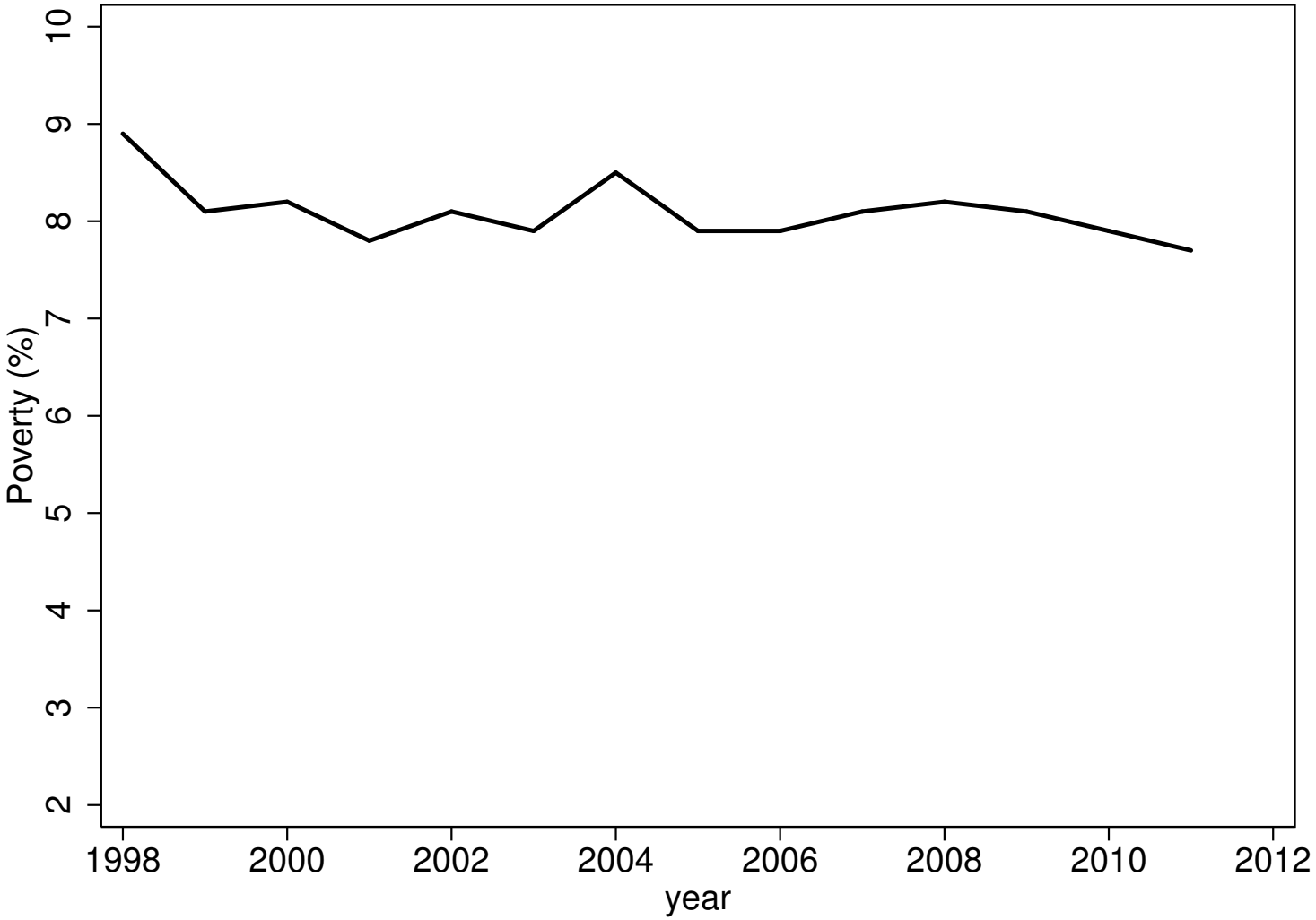
- we need to understand what measure is used,
- what the measure captures, and
- what the measure does not capture

Poverty in Norway, 1998–2012

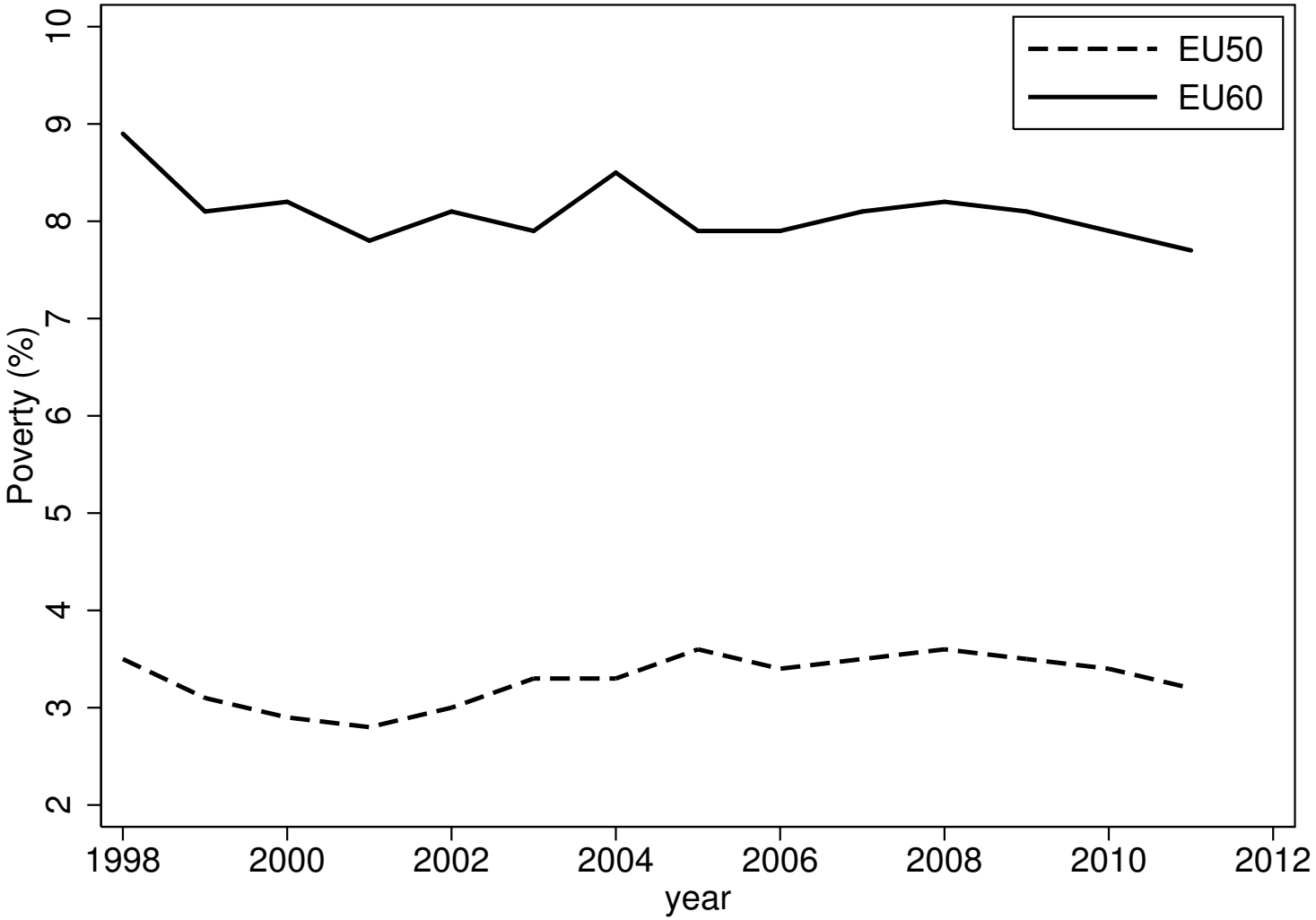
Audun Lysbakken (leader of the Socialist Left Party), said in the 2013 election that

> *"The share of poor people in the Norwegian population is the lowest since 1998."*

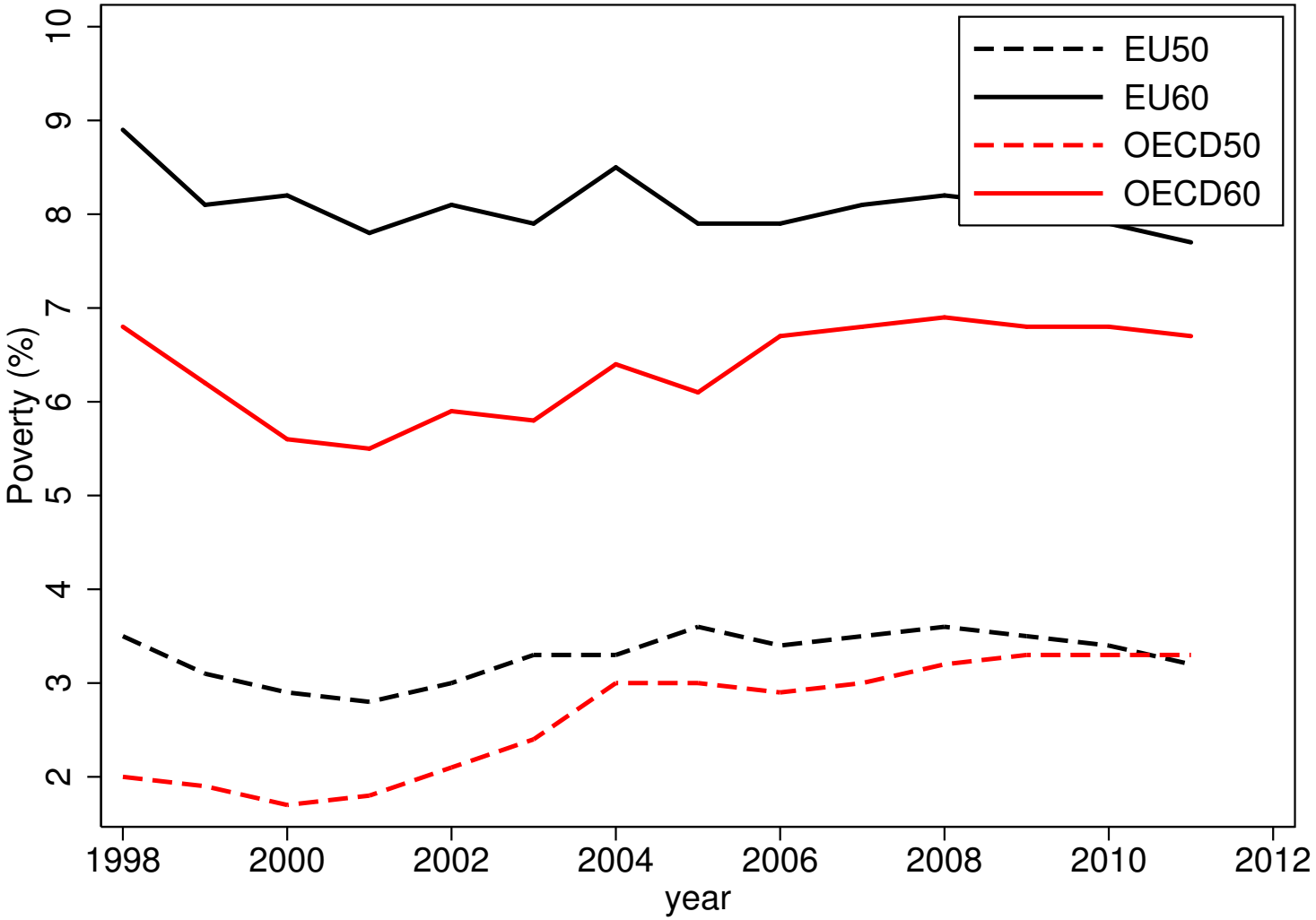
Poverty in Norway, 1998–2012



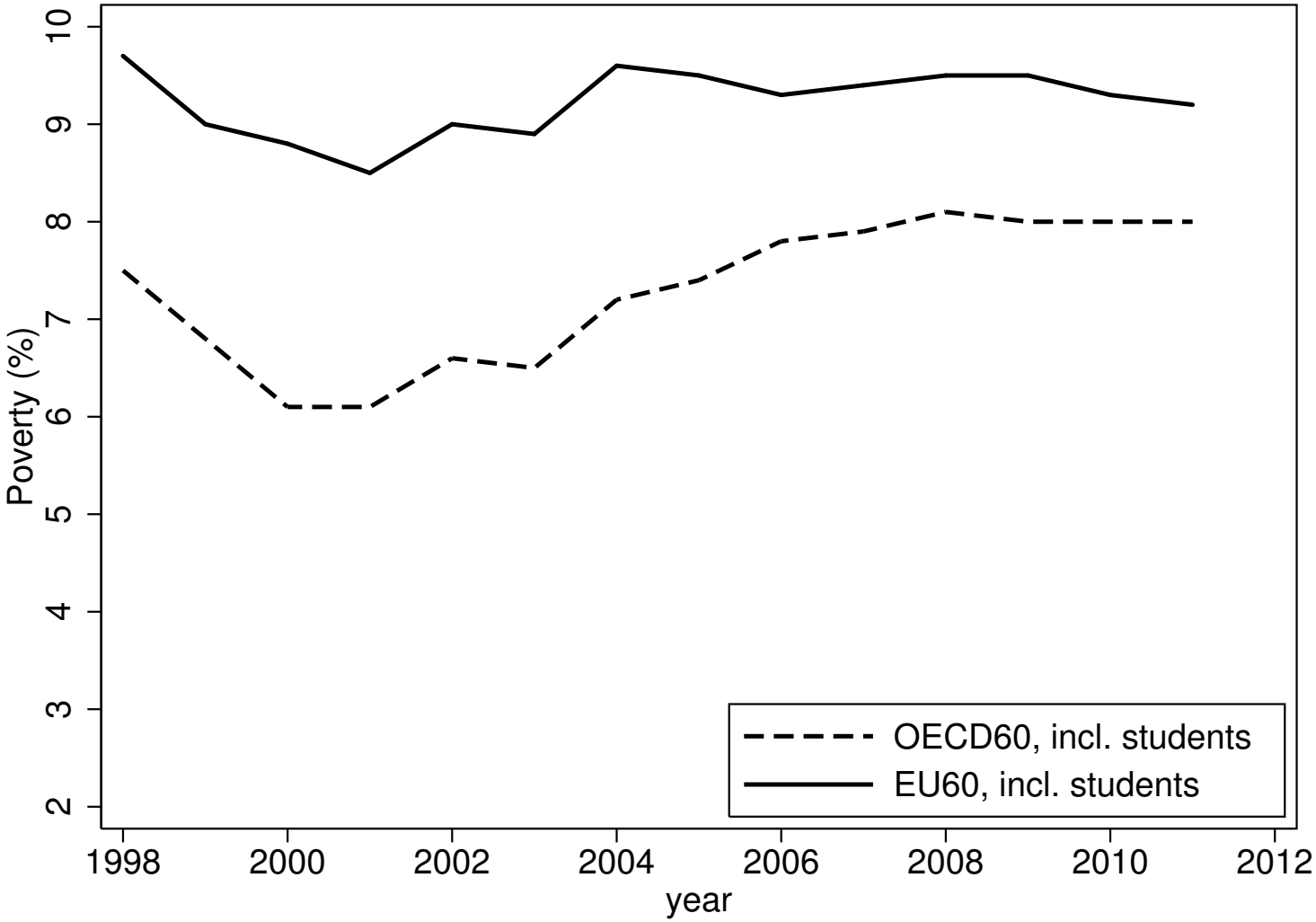
Poverty in Norway, 1998–2012



Poverty in Norway, 1998–2012



Poverty in Norway 1998–2012, incl. students



Plan for today

1. Measures of poverty
   - the income distribution
   - the incidence of poverty
     - poverty lines: absolute poverty vs relative poverty
     - the head count ratio
   - the extent of poverty
     - the poverty gap ratio
     - the income gap ratio
     - Foster–Greer-Thorbecke

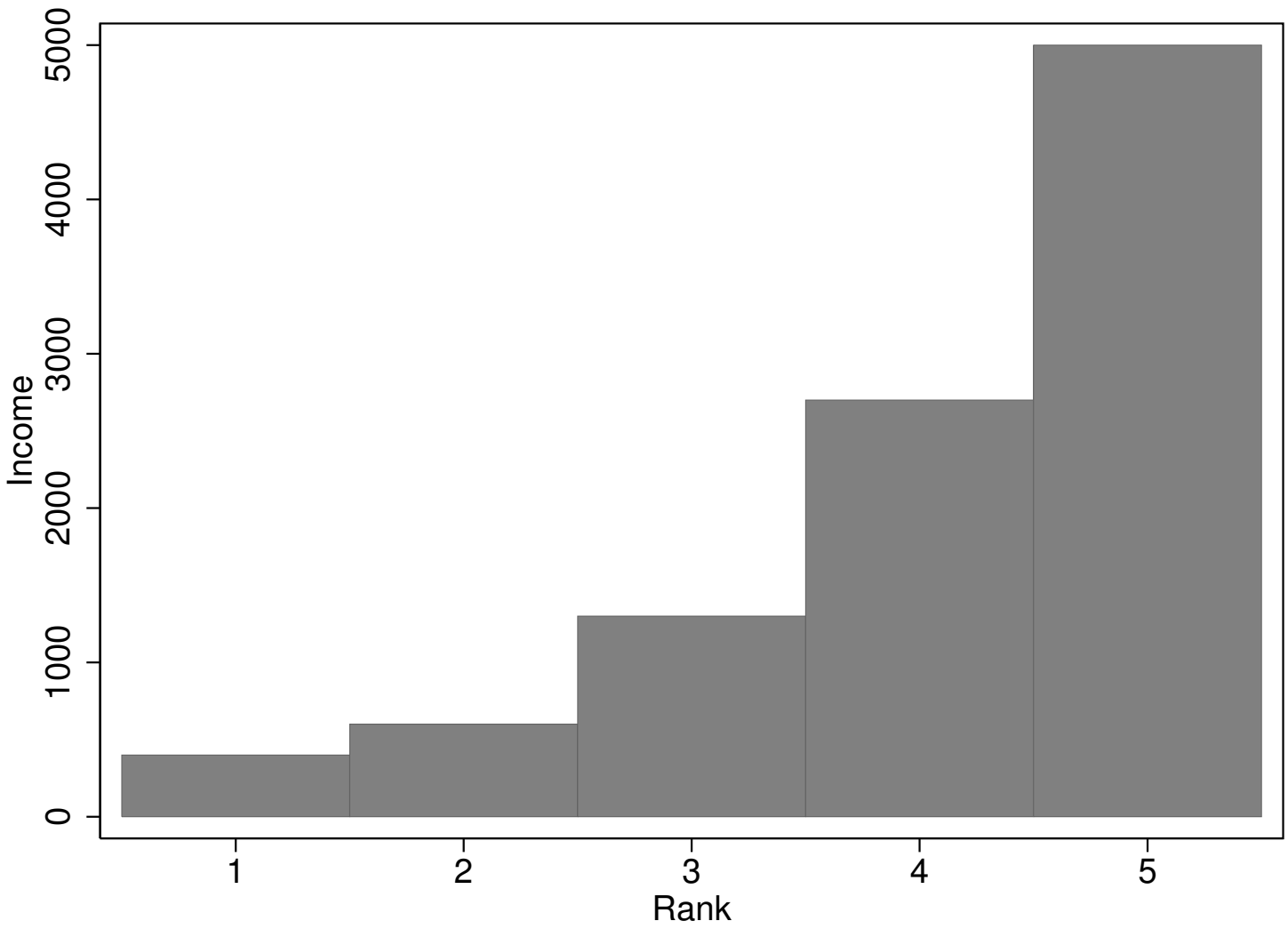
Plan for today

2. Measures of inequality
   - four criteria
   - the Lorenz curve
   - percentile ratios
   - the coefficient of variation
   - the Gini index
3. Some additional issues
   - what income measure?
   - what observation unit?
   - what time frame?

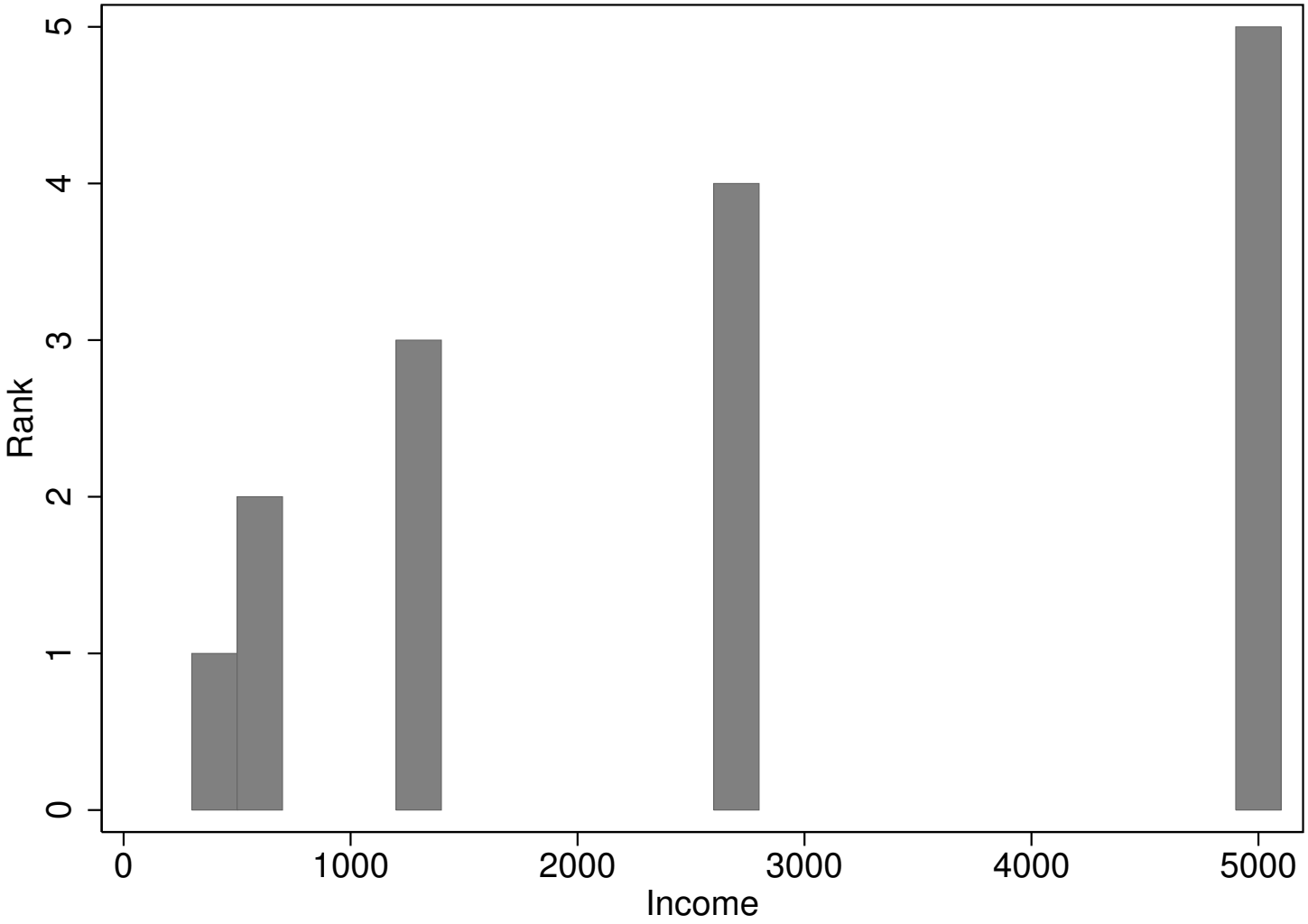
Example

Individual	Adam	John	Emily	Mark	Ted	Total
Income	400	600	1,300	2,700	5,000	10,000
Quintile	1st	2nd	3rd	4th	5th	All
% of income	4	6	13	27	50	100

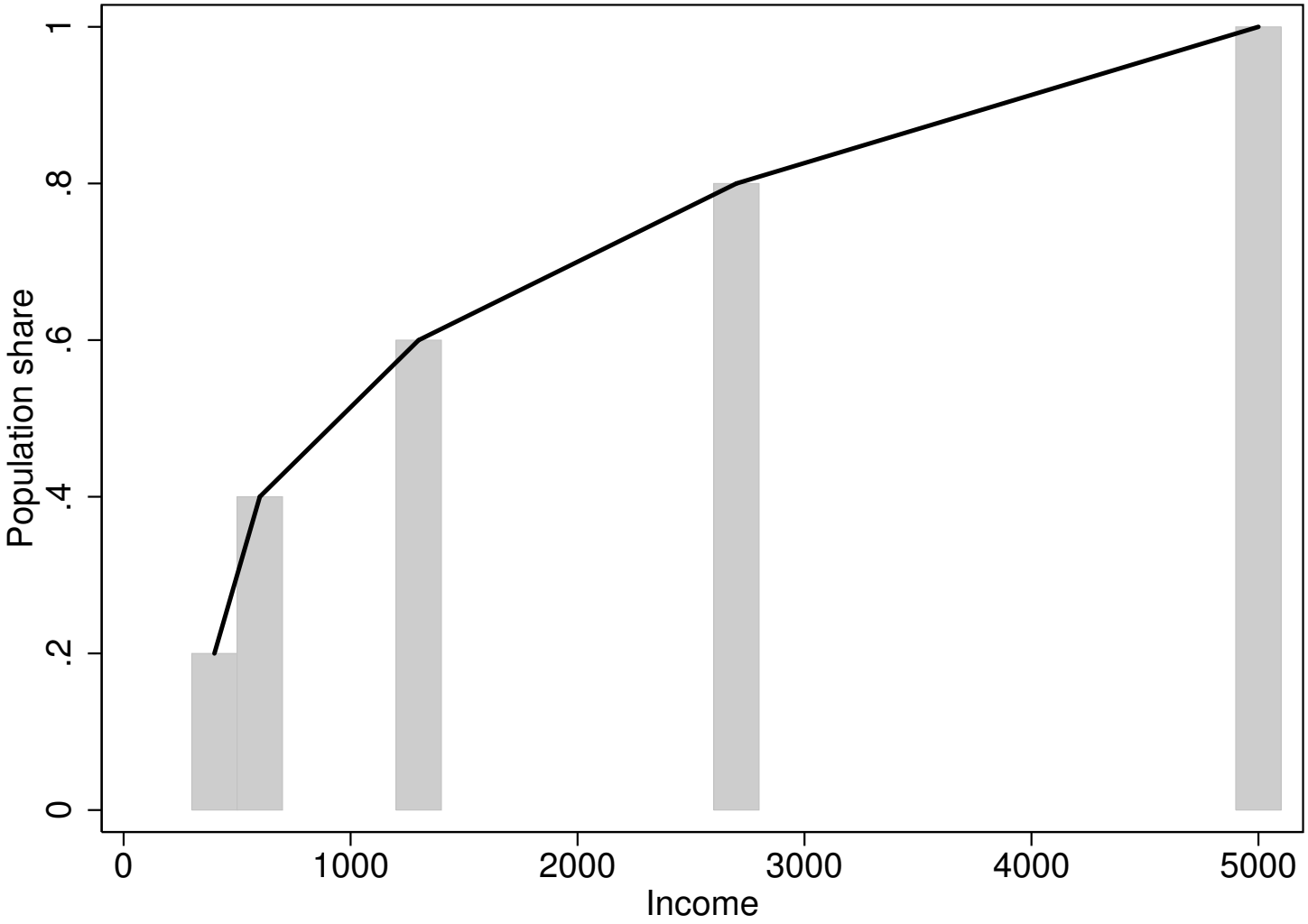
Example: Income by rank



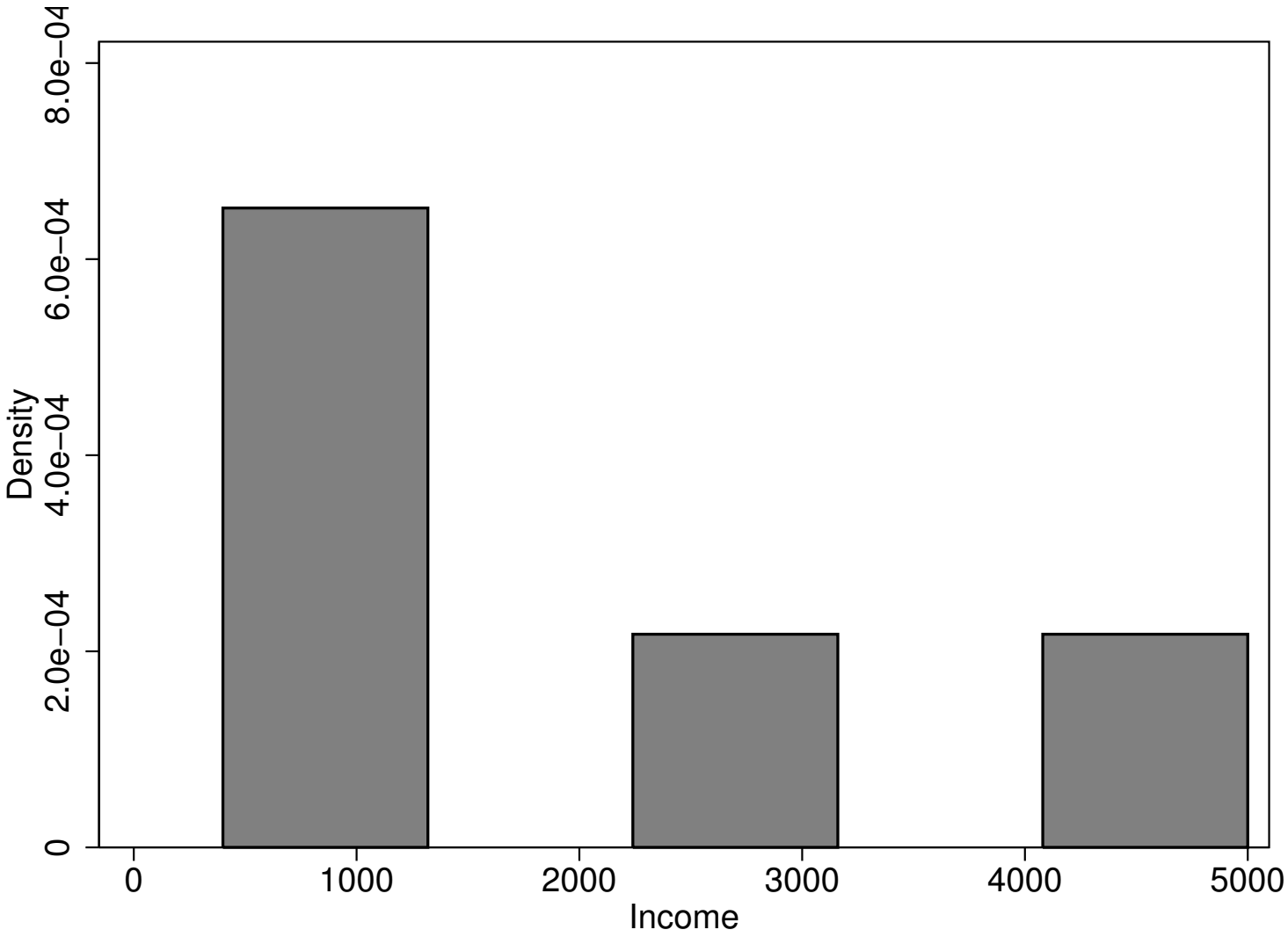
Population share below income levels



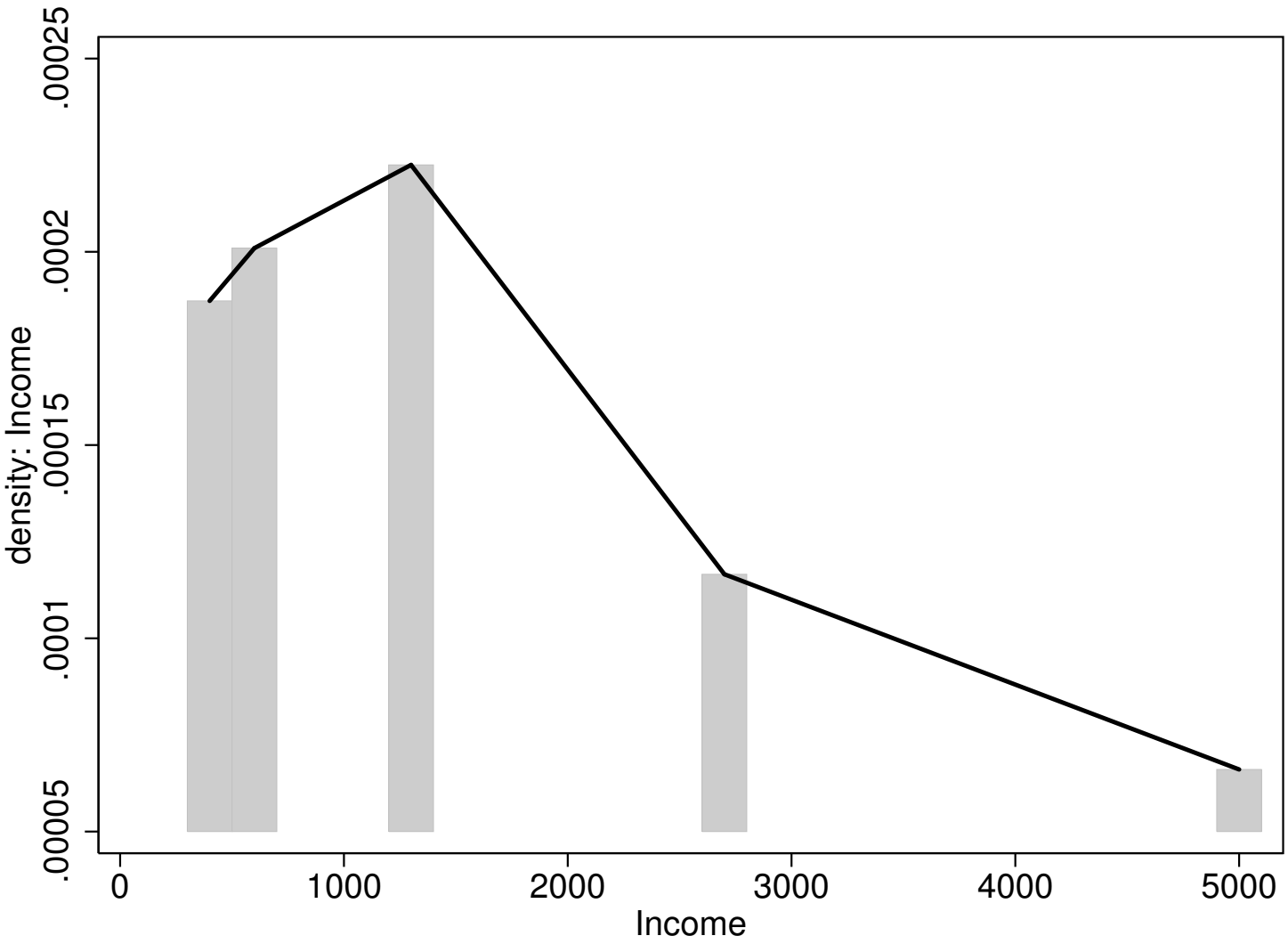
The cumulative density function (CDF)



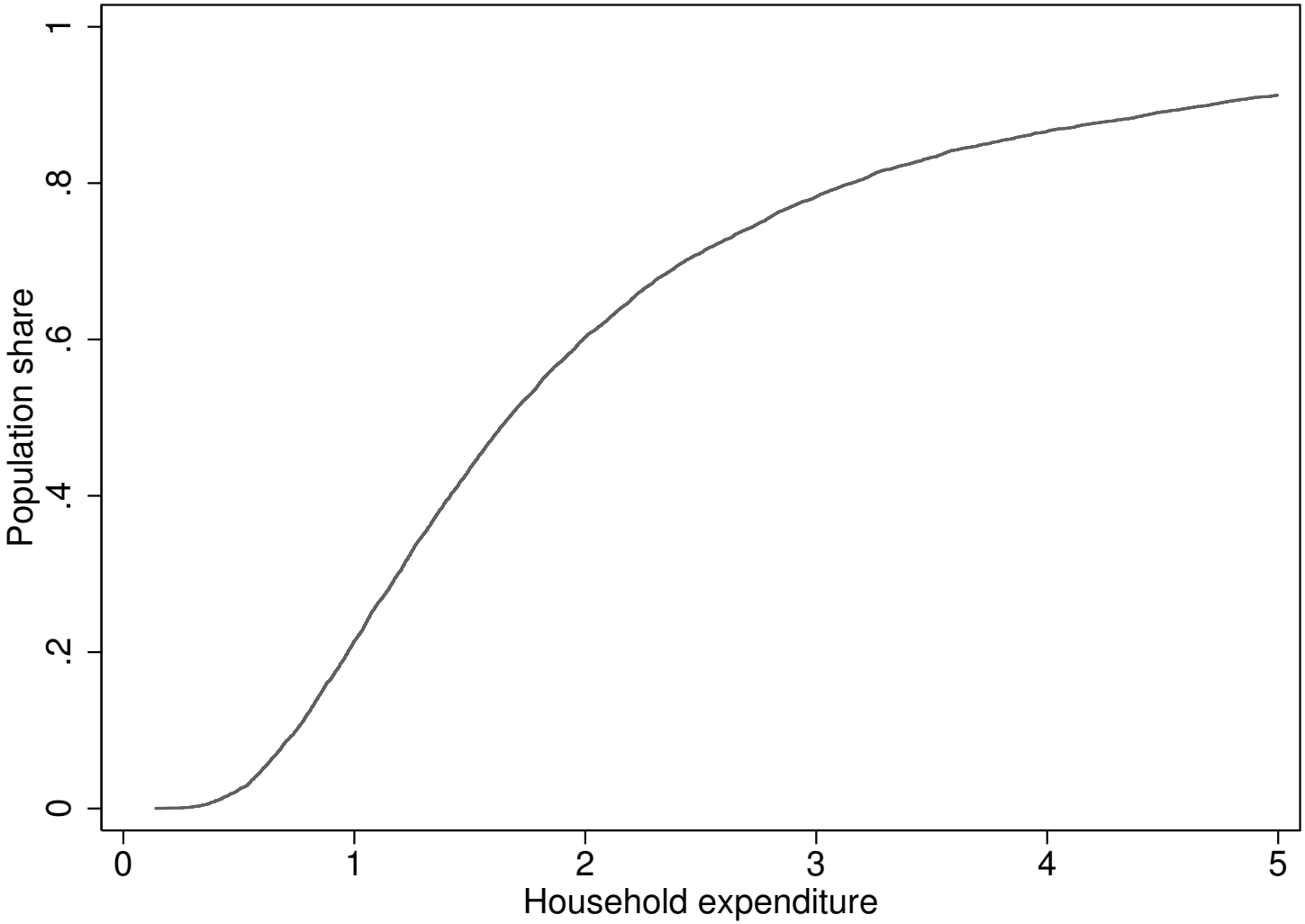
Histogram


Density
0
2.0e−04
4.0e−04
6.0e−04
8.0e−04
0
1000
2000
3000
4000
5000
Income

The probability density function (PDF)



Household expenditure in Uganda, 2006



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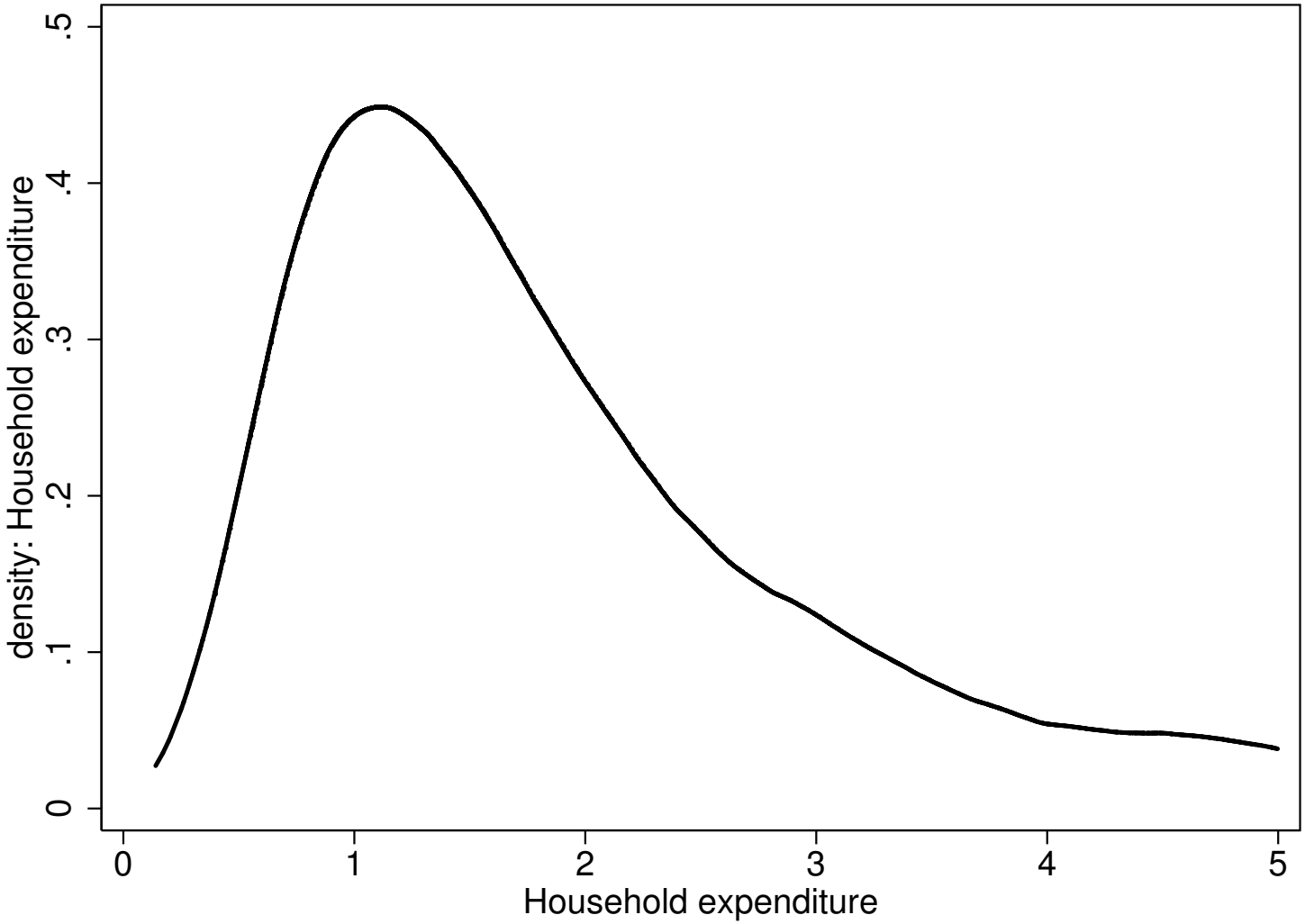


Absolute poverty vs relative poverty

Poverty is defined in either relative or absolute terms

1. Absolute poverty: Income below a fixed limit
   - survival, calories
2. Relative poverty: Income below a limit defined from “normal” income
   - OECD50 vs EU60 (median)

What do you think is the best way to measure poverty?

The incidence of poverty: *HCR*

The head count ratio (*HCR*) is the most common poverty measure

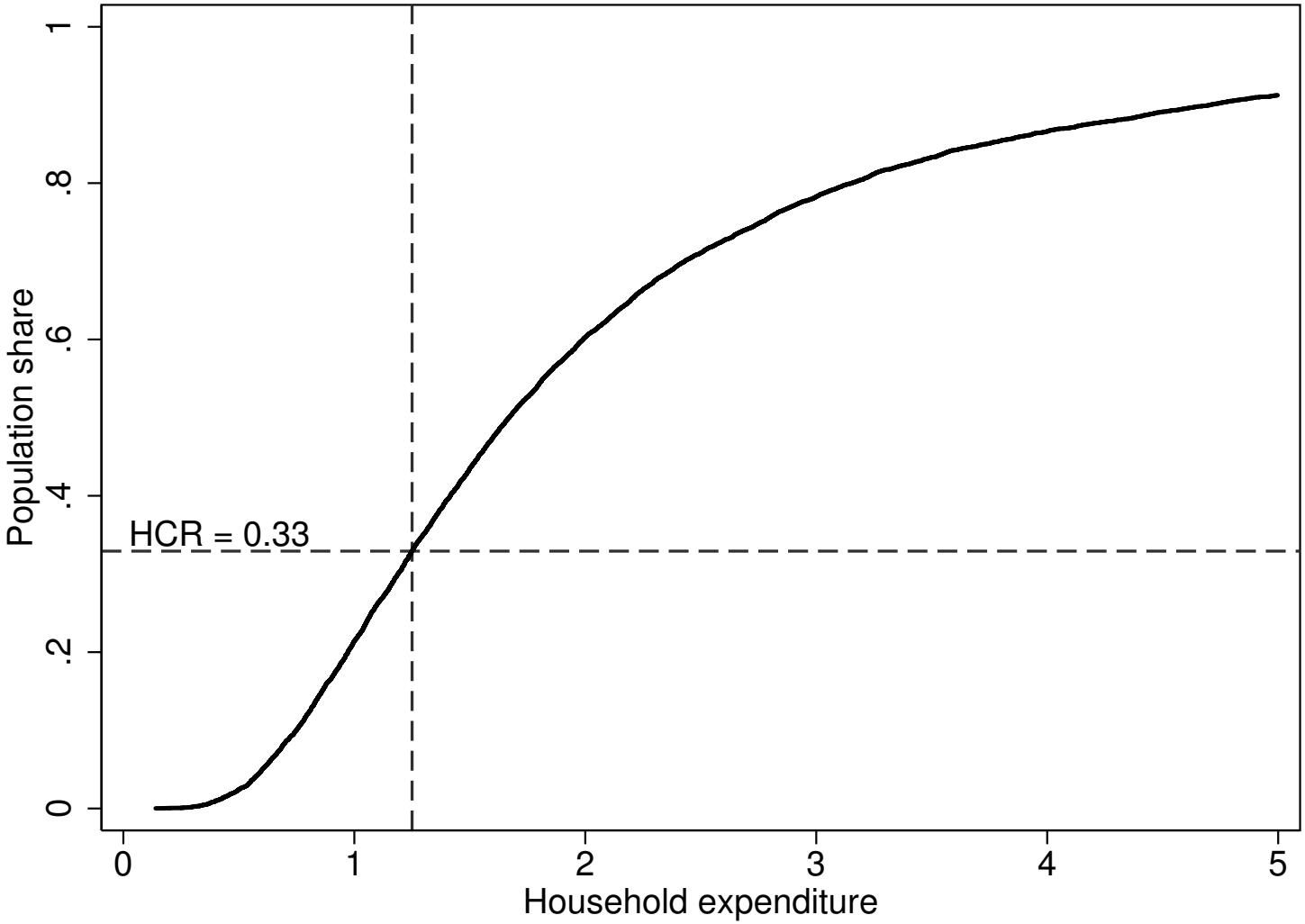
- counts the number of people below a poverty line p
- as a share of the population n

$$HCR = \frac{HC}{n} = \frac{\sum 1\left(y_i < p\right)}{n}$$

This is used to measure both absolute and relative poverty

- Relative: OECD50, EU60
- Absolute: Dollar-a-day measures

HCR in Uganda, 2006



The incidence of poverty: *HCR* – Issues

Key problem: HCR is that it does not take into account the *extent* (or *intensity*) of poverty.

- HCR does not change if the poor's living standards improve if they remain below the poverty line
- the "bang for the buck" problem
- most efficient way of reducing HCR is to help the least poor
- even at the expense of the poorest!

The extent of poverty: *PGR*

The poverty gap ratio (*PGR*)

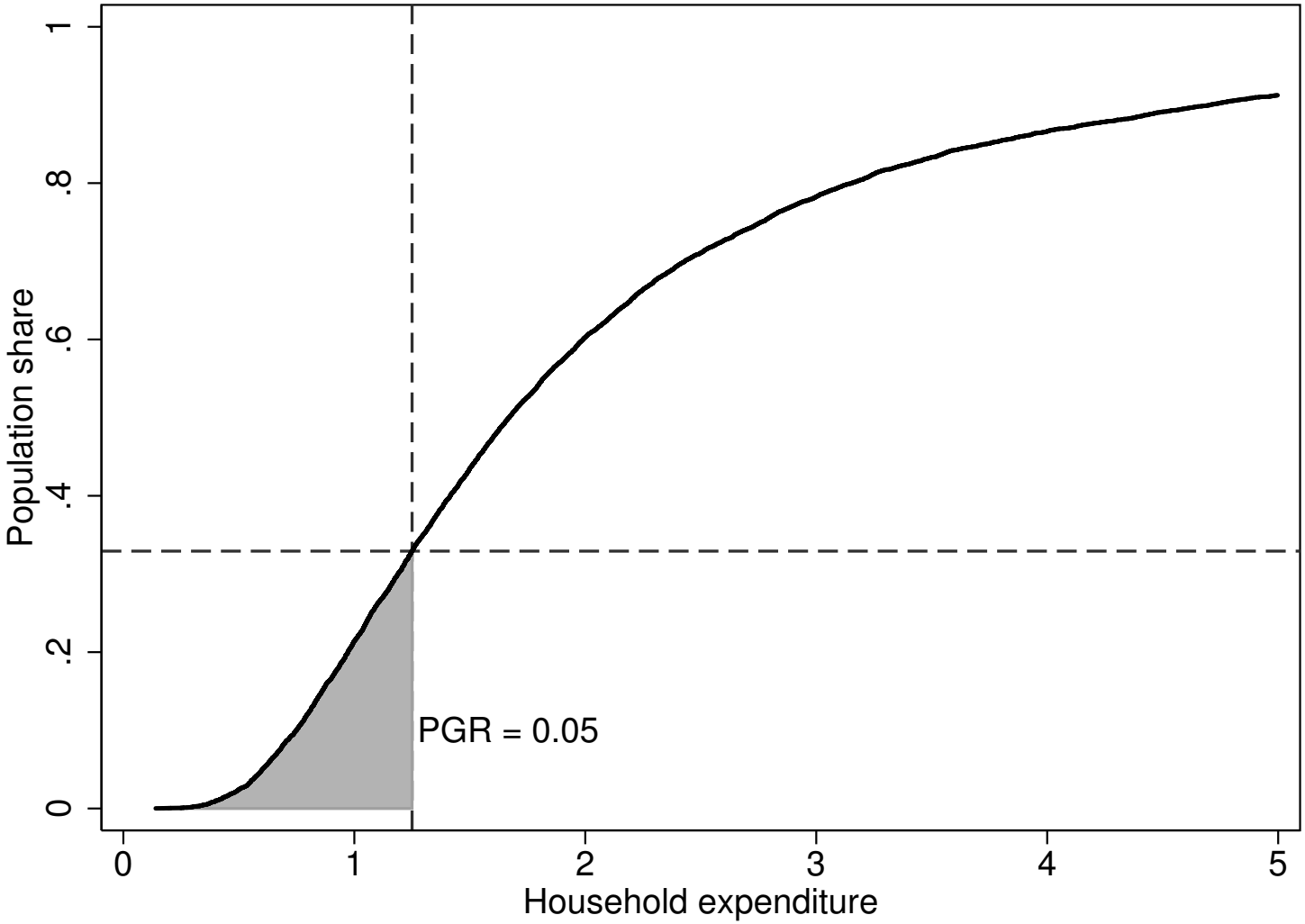
- totals the income short-fall of the poor, $(p - y_i)$
- as a fraction of total income in the population, $n \times m = n\frac{1}{n}\sum_{i=1}^{n} y_i$

$$PGR = \frac{\sum_{i=1}^{HC} (p - y_i)}{nm}$$

where we have ordered incomes so that

$$y_1 \leq y_2 \leq \cdots \leq y_n$$

PGR in Uganda, 2006



The extent of poverty: *PGR* – Issues

PGR can be interpreted as

- the minimum cost of bringing the poor out of poverty
- disregarding implementation costs and distortions

But what if society is highly unequal,

- wealthy overall
- a large number of poor

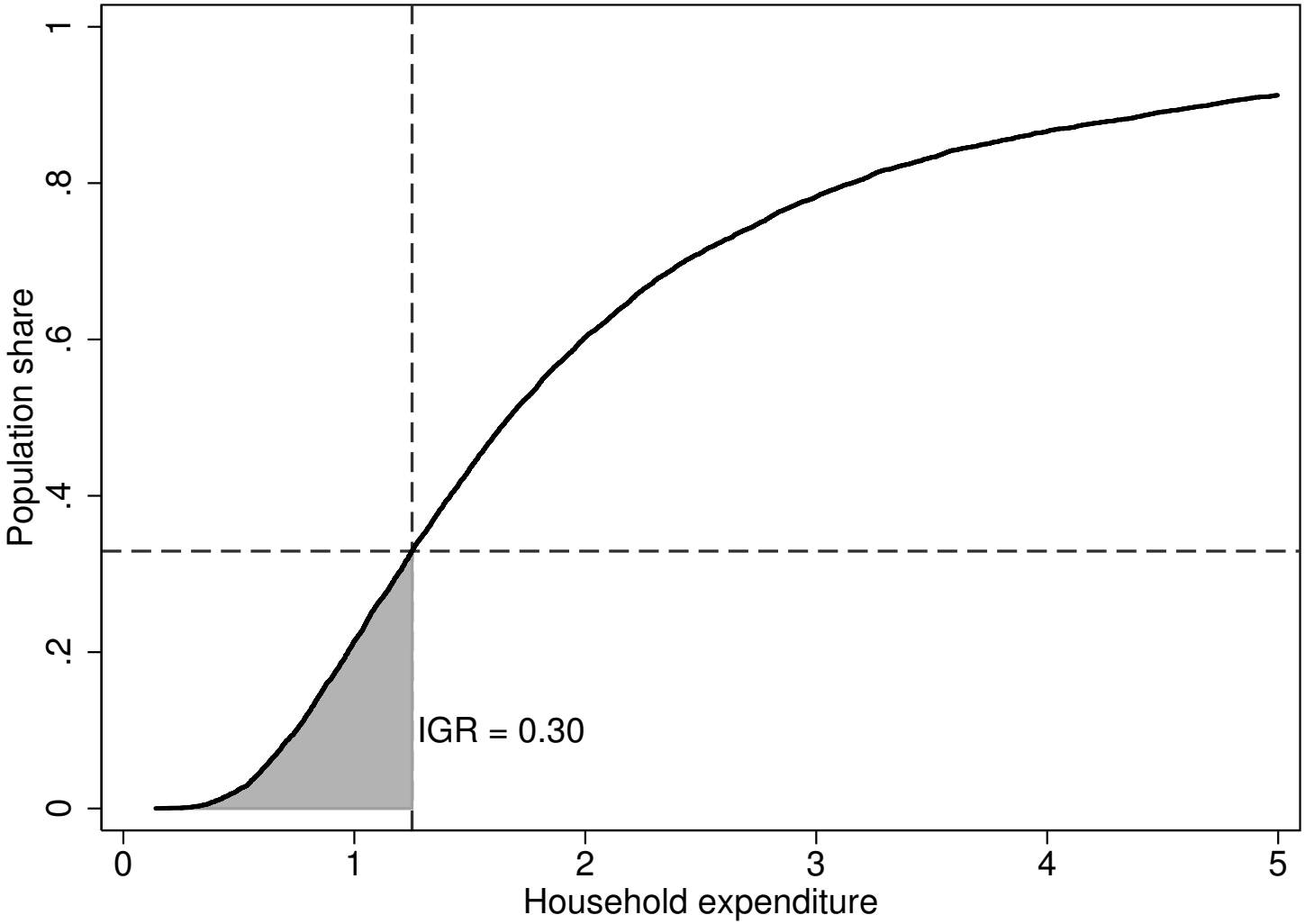
The extent of poverty: *IGR*

The income gap ratio (*IGR*)

- totals the income short-fall of the poor, $(p - y_i)$
- as a fraction of minimum total income if all poor were brought out of poverty, $p \times HC$

$$IGR = \frac{\sum_{i=1}^{HC} (p - y_i)}{p \times HC}$$

IGR in Uganda, 2006



The extent of poverty: *FGT*

All previous measures fail the basic test

- Income transfers from richer to poorer,
- between two poor people,
- should increase poverty

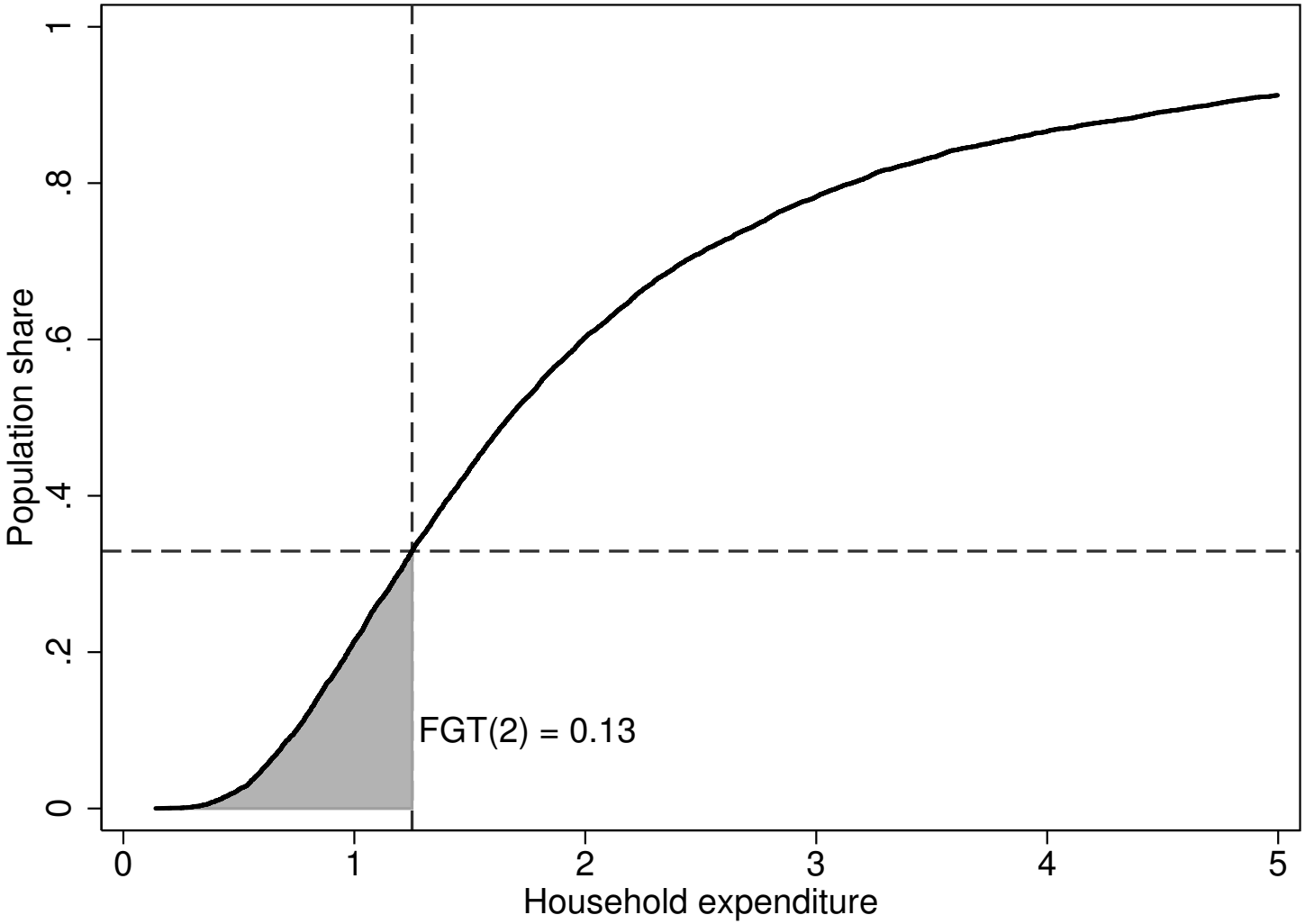
The Foster-Greer-Thorbecke measure (FGT) passes this test, as long as $\alpha > 1$.

$$FGT(\alpha) = \frac{1}{n}\sum_{i=1}^{HC}\left(\frac{p - y_i}{p}\right)^{\alpha}$$

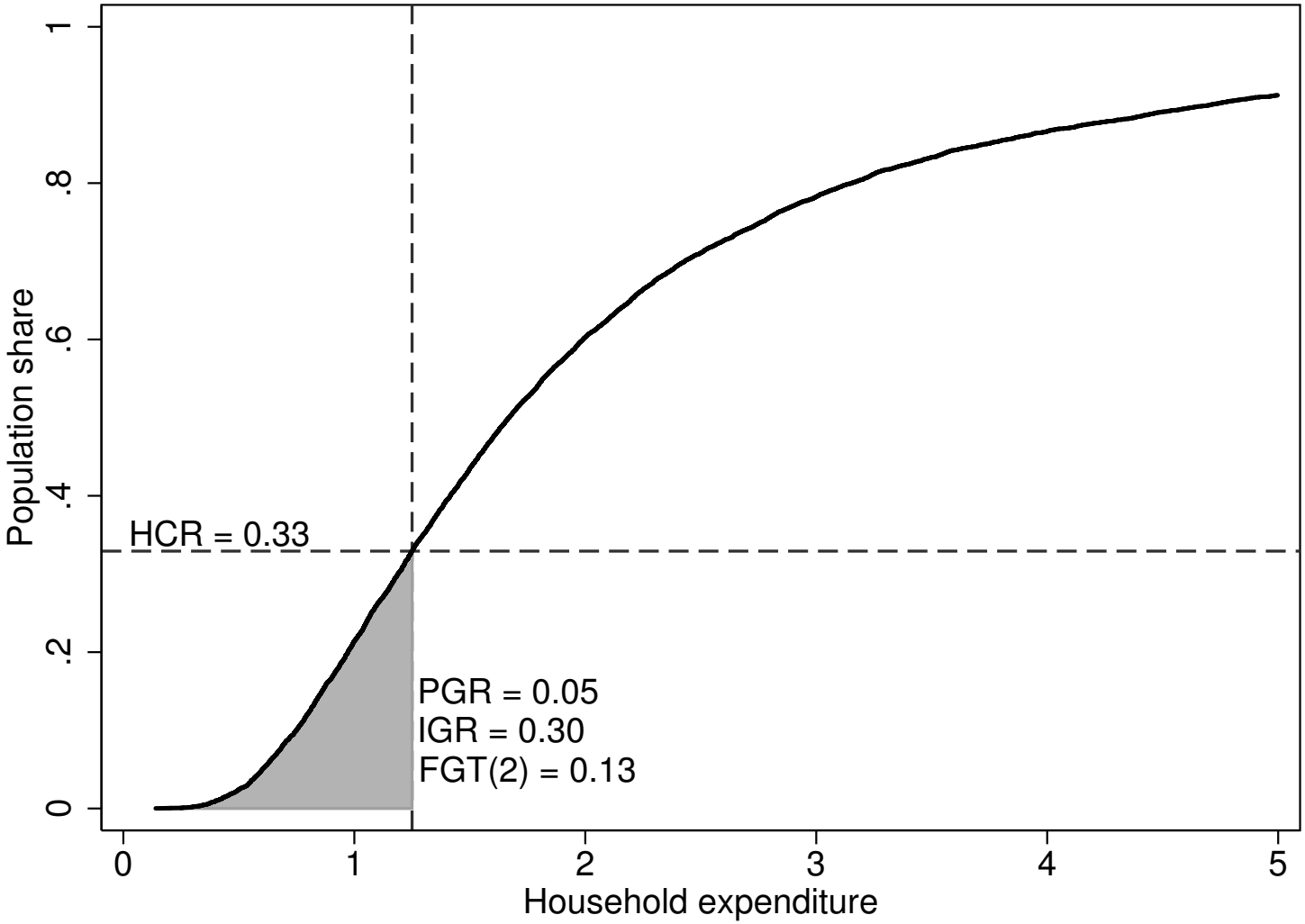
Note also that

$$\begin{aligned} FGT(0) &= HCR \\ FGT(1) &= PGR \end{aligned}$$

FGT in Uganda, 2006 ($\alpha = 2$)



Poverty in Uganda, 2006



Measurement of inequality

Four criteria for inequality measures

1. Anonymity principle
   - *who* earns what does not matter
2. Population principle
   - population *size* does not matter
3. Relative income principle
   - only *relative* income should matter, not income levels
4. The Pigou–Dalton principle
   - if the giver stays poorer than the recipient, then
   - transfer from poorer to (weakly) richer increases inequality

Example

Individual	Adam	John	Emily	Mark	Ted	Total
Income	400	600	1,300	2,700	5,000	10,000
Quintile	1st	2nd	3rd	4th	5th	All
% of income	4	6	13	27	50	100

Anonymity principle

Permutations of the income distribution should not affect inequality

$$I\left(y_1, y_2, \ldots y_{k-1}, y_{k+1}, \ldots, y_n\right) = I\left(y_1, y_2, \ldots y_{k+1}, y_{k-1}, \ldots, y_n\right)$$

Individual	Adam	John	Emily	Mark	Ted	Total
Original						
Income	400	600	1,300	2,700	5,000	10,000
Quintile	1st	2nd	3rd	4th	5th	All
% of income	4	6	13	27	50	100
After permutations						
Income	1,300	400	5000	2,700	600	10,000
Quintile	3rd	1st	5th	4th	2nd	All
% of income	13	4	50	27	6	100

Population principle

The size of the population should not matter

- i.e. if we clone the current distribution, then inequality should be the same

$$I\left(y_1, y_2, \ldots, y_n\right) = I\left(y_1, y_2, \ldots, y_n, y_1, y_2, \ldots, y_n\right)$$

Relative income principle

Only relative incomes should affect inequality

- differences in a rich country should be counted equally as
- differences in a poor country

Individual	Adam	John	Emily	Mark	Ted	Total
Original						
Income	400	600	1,300	2,700	5,000	10,000
Quintile	1st	2nd	3rd	4th	5th	All
% of income	4	6	13	27	50	100
After scaling						
Income	800	1,200	2,600	5,400	10,000	20,000
Quintile	1st	2nd	3rd	4th	5th	All
% of income	4	6	13	27	50	100

Pigou–Dalton principle

If a distribution F can be constructed from another G

- by a set of regressive transfers (poor to rich)
- then inequality is larger in F
- that is, for $h > 0$,

$$\begin{aligned} & I\left(y_1, y_2, \ldots, y_s, \ldots, y_t, \ldots, y_n\right) \\ < & I\left(y_1, y_2, \ldots, y_s - h, \ldots, y_t + h, \ldots, y_n\right) \end{aligned}$$

Pigou–Dalton principle

Note that we should define with regressive transfers, not progressive.

- to see why, consider an egalitarian distribution, where all incomes are equal
- with regressive transfers, indeed

$$I(y, y, \ldots, y, \ldots, y, \ldots, y) < I(y, y, \ldots, y - h, \ldots, y + h, \ldots, y)$$

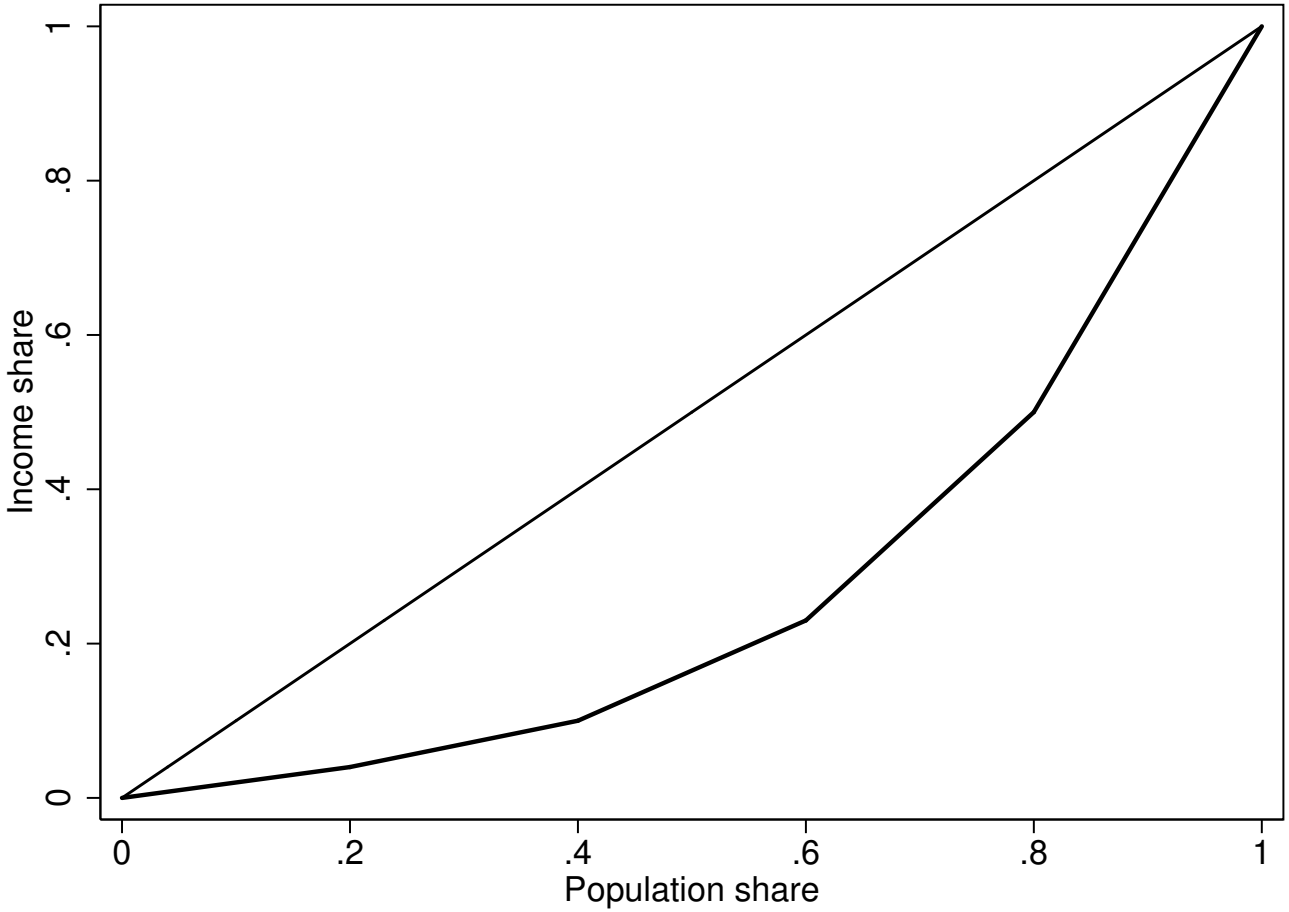
- but with progressive transfers

$$I(y, y, \ldots, y, \ldots, y, \ldots, y) < I(y, y, \ldots, y + h, \ldots, y - h, \ldots, y)!$$

The Lorenz curve

The Lorenz curve summarizes all information necessary for calculating inequality indices under the above criteria

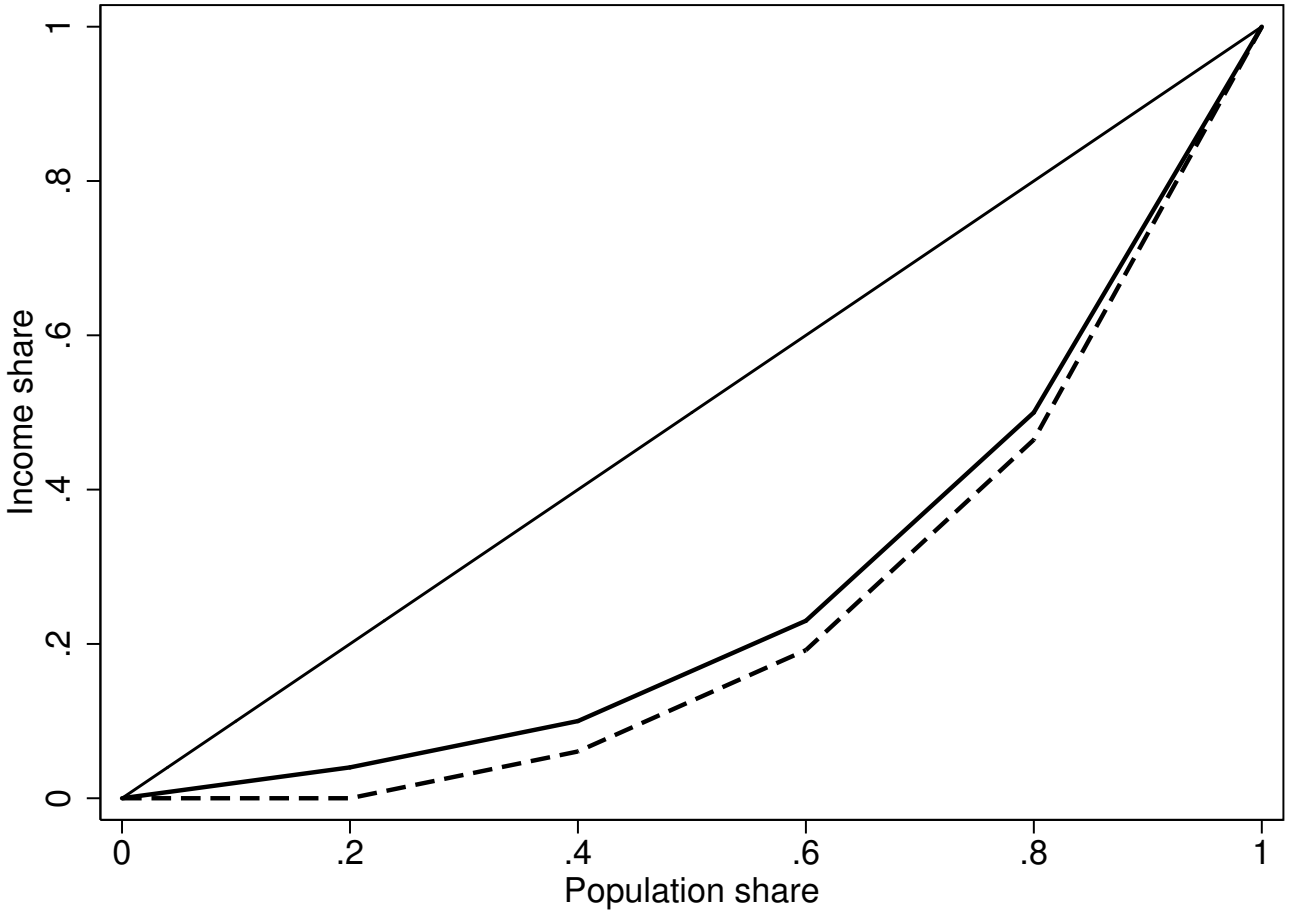
- income shares
- population shares



A regressive transfer

Individual	Adam	John	Emily	Mark	Ted	Total
Income 1	400	600	1,300	2,700	5,000	10,000
Income 2	0	600	1,300	2,700	5,400	10,000

A regressive transfer



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Income 1	400	600	1,300	2,700	5,000	10,000
Income 2	0	600	1,300	2,700	5,400	10,000

Summary measures of inequality

Two problems with the Lorenz curves

1. Often interested in summarizing inequality by a number
2. No ranking when Lorenz curves cross

A summary inequality measure provides a complete ranking

- Different summary measures may disagree in ranking

Percentile ratios (Kuznets ratios)

Percentile ratios take the fraction of percentiles, for instance

$$d9d1 = \frac{p90}{p10}$$
$$d5d1 = \frac{p50}{p10}$$

Easy to calculate and understand, but

- ignores incomes between percentiles
- ignores incomes above the highest and below the lowest
- fails Pigou–Dalton

The coefficient of variation

Takes advantage of the entire income distribution

- Consider income distances from the average income
- giving higher weight to larger deviations

$$CV = \frac{1}{m}\sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_j - m)^2}$$

It satisfies all four principles and is therefore Lorenz-consistent.

The Gini coefficient

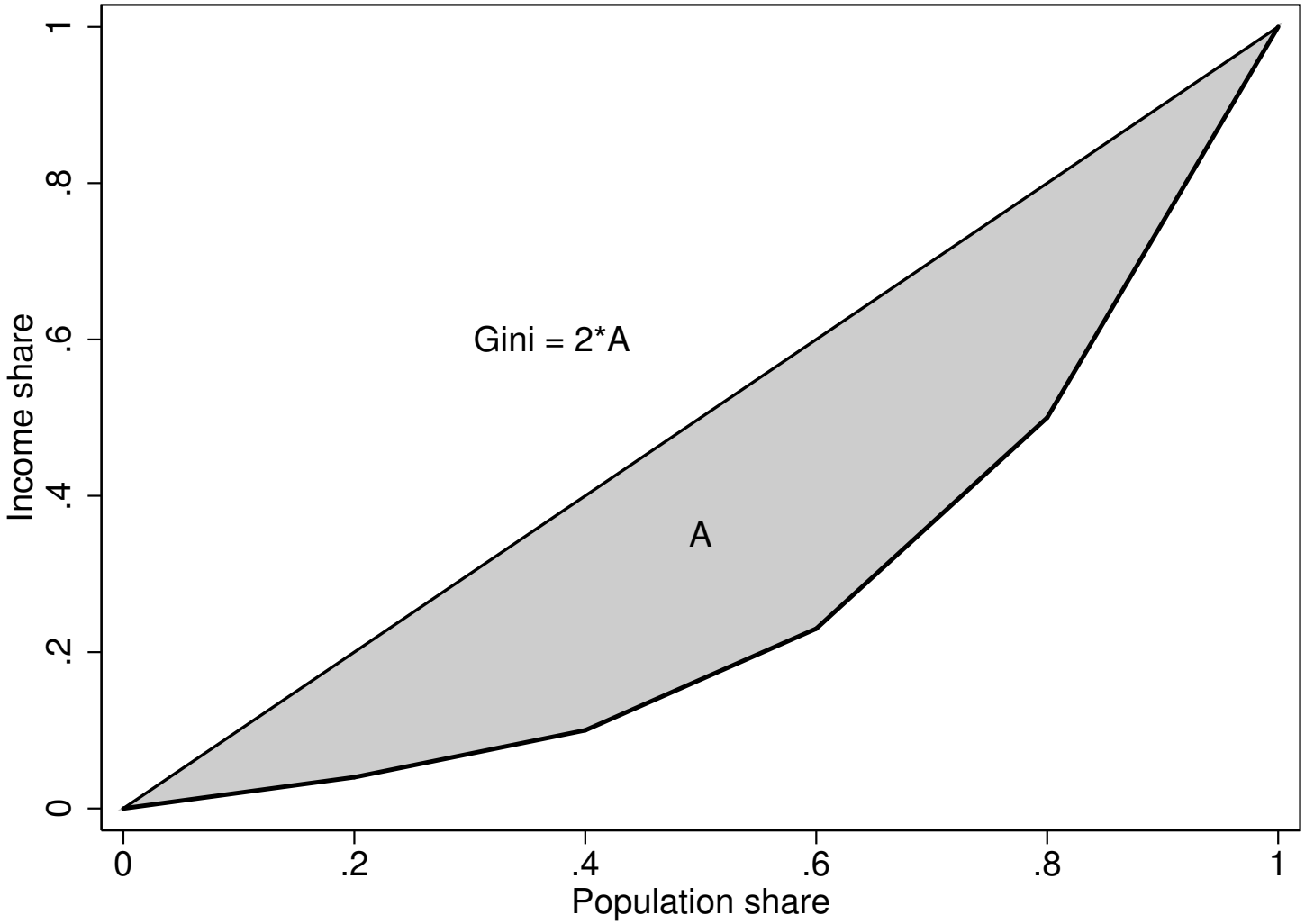
The Gini coefficient is the most used in empirical work.

- Takes the difference between all pairs of incomes and simply totals the (absolute) difference.
- Sequentially, compare an individual *Joe* to all other individuals, $\frac{1}{nm}\sum_{k=1}^{n}|y_{Joe} - y_k|$
- And then do the same for everybody,

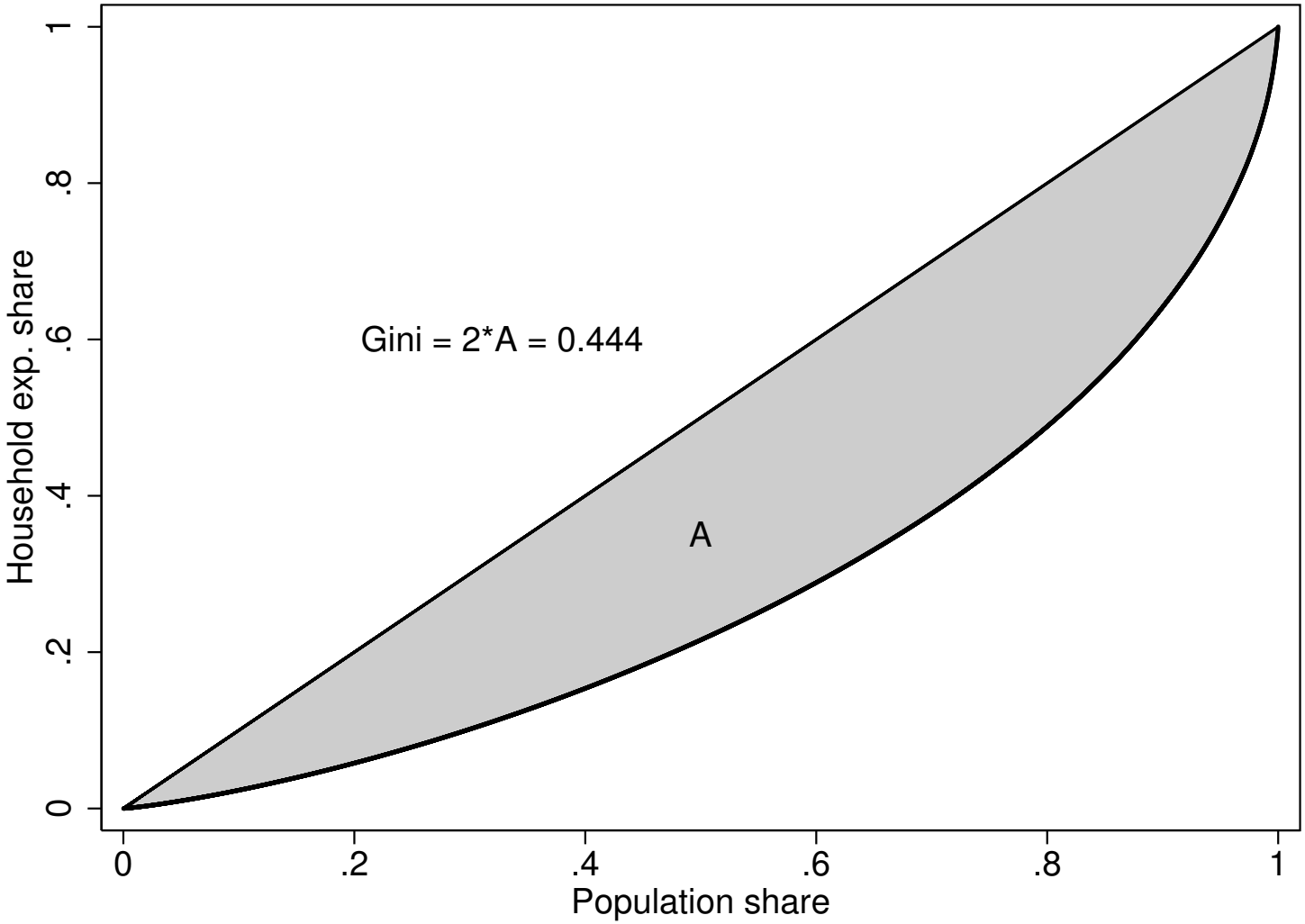
$$G = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{nm}\sum_{k=1}^{n}|y_i - y_k|$$

It satisfies all four principles and is therefore Lorenz-consistent.

Gini coefficient



Lorenz curve and Gini in Uganda, 2006



Comparing Gini and CV

Both satisfy all four principles.

- If Lorenz curves do not cross – always rank the same
- If Lorenz curves do cross – may rank differently

In practice,

- Gini is sensitive to the middle of the distribution
- CV is sensitive to the right tail of the distribution (the rich)
- Neither is very sensitive to the poor

Neither is by any means perfect,

- Gini = 0.5
  - 50% richest share all income equally
  - 75% poorest share 25%, while 25% richest share 75%, both equally

Some additional issues

What is the basis for calculating poverty (or inequality)?

- Income
- Consumption
- Consumption-potential

What is the unit we are interested in?

- Individuals
- Households
- Equivalence scales

Note that we are essentially ignoring distribution inside the household.

Some additional issues

What if high consumption or income today is followed by low consumption or income tomorrow?

- transient poverty vs persistent poverty
- income today vs lifetime income
  - depends on ability to smooth income
  - how reliant are you on variable income (think about agriculture)
  - financial markets: insurance, credit and savings
  - public safety net: unemployment, disability, social assistance, income replacement