Measurement of poverty and inequality

Econ 1910 – Poverty and distribution in developing countries

Tarjei Havnes
Introduction

- How many are poor?
- How large is inequality?
- How did the financial crises affect poverty?

To answer these questions, we need summary measures.

- to understand the present
- to compare over time
- to compare across countries
Introduction

In order to make useful summary measures, we need to specify what we want them to capture.

- How should a poverty measure respond to changes in income
  - among the poor?
  - among the rich?
  - among the middle class?
Introduction

In order to make useful summary measures, we need to specify what we want them to capture.

▶ How should a poverty measure respond to changes in income

   ▶ among the poor?
   ▶ among the rich?
   ▶ among the middle class?

▶ How should an inequality measure respond to these changes?
In order to understand statements like “poverty has decreased”

- we need to understand what measure is used,
- what the measure captures, and
- what the measure does not capture
Poverty in Norway, 1998–2012

Audun Lysbakken (leader of the Socialist Left Party), said in the 2013 election that

“The share of poor people in the Norwegian population is the lowest since 1998.”
Poverty in Norway, 1998–2012

![Graph showing poverty percentage from 1998 to 2012. The graph indicates a general decline in poverty rates over the years, with some fluctuations.](image-url)
Poverty in Norway, 1998–2012
Poverty in Norway, 1998–2012

[Graph showing poverty percentages over the years for EU50, EU60, OECD50, and OECD60 categories.]
Poverty in Norway 1998–2012, incl. students

![Graph showing poverty percentages over years for OECD60 and EU60, including students.](image_url)

- **OECD60, incl. students**
- **EU60, incl. students**
Plan for today

1. Measures of poverty
   ▶ the income distribution
   ▶ the incidence of poverty
     ▶ poverty lines: absolute poverty vs relative poverty
     ▶ the head count ratio
   ▶ the extent of poverty
     ▶ the poverty gap ratio
     ▶ the income gap ratio
     ▶ Foster–Greer-Thorbecke
Plan for today

2. Measures of inequality
   ◀ four criteria
   ◀ the Lorenz curve
   ◀ percentile ratios
   ◀ the coefficient of variation
   ◀ the Gini index

3. Some additional issues
   ◀ what income measure?
   ◀ what observation unit?
   ◀ what time frame?
<table>
<thead>
<tr>
<th>Individual</th>
<th>Adam</th>
<th>John</th>
<th>Emily</th>
<th>Mark</th>
<th>Ted</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>400</td>
<td>600</td>
<td>1,300</td>
<td>2,700</td>
<td>5,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Quintile</td>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
<td>4th</td>
<td>5th</td>
<td>All</td>
</tr>
<tr>
<td>% of income</td>
<td>4</td>
<td>6</td>
<td>13</td>
<td>27</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>
Example: Income by rank
Population share below income levels
The cumulative density function (CDF)
The probability density function (PDF)
Household expenditure in Uganda, 2006
Household expenditure in Uganda, 2006
Absolute poverty vs relative poverty

Poverty is defined in either relative or absolute terms

1. Absolute poverty: Income below a fixed limit
   - survival, calories

2. Relative poverty: Income below a limit defined from “normal” income
   - OECD50 vs EU60 (median)

What do you think is the best way to measure poverty?
The incidence of poverty: \textit{HCR}

The head count ratio (\textit{HCR}) is the most common poverty measure

- counts the number of people below a poverty line \( p \)
- as a share of the population \( n \)

\[
HCR = \frac{HC}{n} = \frac{\sum 1(y_i < p)}{n}
\]

This is used to measure both absolute and relative poverty

- Relative: OECD50, EU60
- Absolute: Dollar-a-day measures
HCR in Uganda, 2006

![Graph showing the relationship between household expenditure and population share with an HCR of 0.33]
The incidence of poverty: *HCR* – Issues

Key problem: HCR is that it does not take into account the *extent* (or *intensity*) of poverty.

- HCR does not change if the poor’s living standards improve if they remain below the poverty line
- the “bang for the buck” problem
- most efficient way of reducing HCR is to help the least poor
- even at the expense of the poorest!
The extent of poverty: \( PGR \)

The poverty gap ratio \( (PGR) \)

- totals the income short-fall of the poor, \( (p - y_i) \)
- as a fraction of total income in the population,

\[
n \times m = n \frac{1}{n} \sum_{i=1}^{n} y_i
\]

\[
PGR = \frac{\sum_{i=1}^{HC} (p - y_i)}{nm}
\]

where we have ordered incomes so that

\[
y_1 \leq y_2 \leq \cdots \leq y_n
\]
PGR in Uganda, 2006

![Graph showing population share vs. household expenditure with PGR = 0.05 shaded area.](image-url)
The extent of poverty: *PGR – Issues*

PGR can be interpreted as

- the minimum cost of bringing the poor out of poverty
- disregarding implementation costs and distortions

But what if society is highly unequal,

- wealthy overall
- a large number of poor
The extent of poverty: *IGR*

The income gap ratio (*IGR*)

- totals the income short-fall of the poor, \((p - y_i)\)
- as a fraction of minimum total income if all poor were brought out of poverty, \(p \times HC\)

\[
IGR = \frac{\sum_{i=1}^{HC} (p - y_i)}{p \times HC}
\]
IGR in Uganda, 2006

Household expenditure vs Population share graph with IGR = 0.30.
The extent of poverty: \textit{FGT}

All previous measures fail the basic test

- Income transfers from richer to poorer,
- between two poor people,
- should increase poverty

The Foster-Greer-Thorbecke measure (FGT) passes this test, as long as $\alpha > 1$.

\[
\text{FGT} (\alpha) = \frac{1}{n} \sum_{i=1}^{HC} \left( \frac{p - y_i}{p} \right)^\alpha
\]

Note also that

$\text{FGT} (0) = HCR$

$\text{FGT} (1) = PGR$
FGT in Uganda, 2006 ($\alpha = 2$)

\[
\text{FGT}(2) = 0.13
\]
Poverty in Uganda, 2006

HCR = 0.33
PGR = 0.05
IGR = 0.30
FGT(2) = 0.13
Measurement of inequality

Four criteria for inequality measures

1. Anonymity principle
   ▶ *who* earns what does not matter

2. Population principle
   ▶ population *size* does not matter

3. Relative income principle
   ▶ only *relative* income should matter, not income levels

4. The Pigou–Dalton principle
   ▶ if the giver stays poorer than the recipient, then
   ▶ transfer from poorer to (weakly) richer increases inequality
## Example

<table>
<thead>
<tr>
<th>Individual</th>
<th>Adam</th>
<th>John</th>
<th>Emily</th>
<th>Mark</th>
<th>Ted</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>400</td>
<td>600</td>
<td>1,300</td>
<td>2,700</td>
<td>5,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Quintile</td>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
<td>4th</td>
<td>5th</td>
<td>All</td>
</tr>
<tr>
<td>% of income</td>
<td>4</td>
<td>6</td>
<td>13</td>
<td>27</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>
Anonymity principle

Permutations of the income distribution should not affect inequality

\[ I(y_1, y_2, \ldots y_{k-1}, y_{k+1}, \ldots, y_n) = I(y_1, y_2, \ldots y_{k+1}, y_{k-1}, \ldots, y_n) \]

<table>
<thead>
<tr>
<th>Individual</th>
<th>Adam</th>
<th>John</th>
<th>Emily</th>
<th>Mark</th>
<th>Ted</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Original</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>400</td>
<td>600</td>
<td>1,300</td>
<td>2,700</td>
<td>5,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Quintile</td>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
<td>4th</td>
<td>5th</td>
<td>All</td>
</tr>
<tr>
<td>% of income</td>
<td>4</td>
<td>6</td>
<td>13</td>
<td>27</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td><strong>After permutations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>1,300</td>
<td>400</td>
<td>5,000</td>
<td>2,700</td>
<td>600</td>
<td>10,000</td>
</tr>
<tr>
<td>Quintile</td>
<td>3rd</td>
<td>1st</td>
<td>5th</td>
<td>4th</td>
<td>2nd</td>
<td>All</td>
</tr>
<tr>
<td>% of income</td>
<td>13</td>
<td>4</td>
<td>50</td>
<td>27</td>
<td>6</td>
<td>100</td>
</tr>
</tbody>
</table>
Population principle

The size of the population should not matter

- i.e. if we clone the current distribution, then inequality should be the same

\[ I(y_1, y_2, \ldots, y_n) = I(y_1, y_2, \ldots, y_n, y_1, y_2, \ldots, y_n) \]
Relative income principle

Only relative incomes should affect inequality

- differences in a rich country should be counted equally as
- differences in a poor country

<table>
<thead>
<tr>
<th>Individual</th>
<th>Adam</th>
<th>John</th>
<th>Emily</th>
<th>Mark</th>
<th>Ted</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td>400</td>
<td>600</td>
<td>1,300</td>
<td>2,700</td>
<td>5,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Quintile</td>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
<td>4th</td>
<td>5th</td>
<td>All</td>
</tr>
<tr>
<td>% of income</td>
<td>4</td>
<td>6</td>
<td>13</td>
<td>27</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>After scaling</td>
<td>800</td>
<td>1,200</td>
<td>2,600</td>
<td>5,400</td>
<td>10,000</td>
<td>20,000</td>
</tr>
<tr>
<td>Quintile</td>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
<td>4th</td>
<td>5th</td>
<td>All</td>
</tr>
<tr>
<td>% of income</td>
<td>4</td>
<td>6</td>
<td>13</td>
<td>27</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>
Pigou–Dalton principle

If a distribution $F$ can be constructed from another $G$

- by a set of regressive transfers (poor to rich)
- then inequality is larger in $F$
- that is, for $h > 0$,

$$I(y_1, y_2, \ldots, y_s, \ldots, y_t, \ldots, y_n) < I(y_1, y_2, \ldots, y_s - h, \ldots, y_t + h, \ldots, y_n)$$
Pigou–Dalton principle

Note that we should define with regressive transfers, not progressive.

► to see why, consider an egalitarian distribution, where all incomes are equal
► with regressive transfers, indeed

\[ I(y, y, \ldots, y, \ldots, y) < I(y, y, \ldots, y - h, \ldots, y + h, \ldots, y) \]

► but with progressive transfers

\[ I(y, y, \ldots, y, \ldots, y) < I(y, y, \ldots, y + h, \ldots, y - h, \ldots, y) \]
The Lorenz curve

The Lorenz curve summarizes all information necessary for calculating inequality indices under the above criteria

- income shares
- population shares
A regressive transfer

<table>
<thead>
<tr>
<th>Individual</th>
<th>Adam</th>
<th>John</th>
<th>Emily</th>
<th>Mark</th>
<th>Ted</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income 1</td>
<td>400</td>
<td>600</td>
<td>1,300</td>
<td>2,700</td>
<td>5,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Income 2</td>
<td>0</td>
<td>600</td>
<td>1,300</td>
<td>2,700</td>
<td>5,400</td>
<td>10,000</td>
</tr>
</tbody>
</table>
A regressive transfer

<table>
<thead>
<tr>
<th>Individual</th>
<th>Adam</th>
<th>John</th>
<th>Emily</th>
<th>Mark</th>
<th>Ted</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income 1</td>
<td>400</td>
<td>600</td>
<td>1,300</td>
<td>2,700</td>
<td>5,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Income 2</td>
<td>0</td>
<td>600</td>
<td>1,300</td>
<td>2,700</td>
<td>5,400</td>
<td>10,000</td>
</tr>
</tbody>
</table>
Summary measures of inequality

Two problems with the Lorenz curves

1. Often interested in summarizing inequality by a number
2. No ranking when Lorenz curves cross

A summary inequality measure provides a complete ranking

- Different summary measures may disagree in ranking
Percentile ratios (Kuznets ratios)

Percentile ratios take the fraction of percentiles, for instance

\[ d_{91} = \frac{p_{90}}{p_{10}} \]
\[ d_{51} = \frac{p_{50}}{p_{10}} \]

Easy to calculate and understand, but

- ignores incomes between percentiles
- ignores incomes above the highest and below the lowest
- fails Pigou–Dalton
The coefficient of variation

Takes advantage of the entire income distribution

▶ Consider income distances from the average income
▶ giving higher weight to larger deviations

\[ CV = \frac{1}{m} \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_j - m)^2} \]

It satisfies all four principles and is therefore Lorenz-consistent.
The Gini coefficient

The Gini coefficient is the most used in empirical work.

- Takes the difference between all pairs of incomes and simply totals the (absolute) difference.
- Sequentially, compare an individual Joe to all other individuals,
  \[ \frac{1}{nm} \sum_{k=1}^{n} |y_{Joe} - y_k| \]
- And then do the same for everybody,
  \[ G = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{nm} \sum_{k=1}^{n} |y_i - y_k| \]

It satisfies all four principles and is therefore Lorenz-consistent.
Gini coefficient

\[ Gini = 2A \]

Graph showing the relationship between income share and population share with the Gini coefficient equation overlayed.
Lorenz curve and Gini in Uganda, 2006

Gini = 2*A = 0.444

Household exp. share

Population share
Comparing Gini and CV

Both satisfy all four principles.

▶ If Lorenz curves do not cross – always rank the same
▶ If Lorenz curves do cross – may rank differently

In practice,

▶ Gini is sensitive to the middle of the distribution
▶ CV is sensitive to the right tail of the distribution (the rich)
▶ Neither is very sensitive to the poor

Neither is by any means perfect,

▶ Gini = 0.5
  ▶ 50% richest share all income equally
  ▶ 75% poorest share 25%, while 25% richest share 75%, both equally
Some additional issues

What is the basis for calculating poverty (or inequality)?

- Income
- Consumption
- Consumption-potential

What is the unit we are interested in?

- Individuals
- Households
- Equivalence scales

Note that we are essentially ignoring distribution inside the household.
Some additional issues

What if high consumption or income today is followed by low consumption or income tomorrow?

- transient poverty vs persistent poverty
- income today vs lifetime income
  - depends on ability to smooth income
  - how reliant are you on variable income (think about agriculture)
  - financial markets: insurance, credit and savings
  - public safety net: unemployment, disability, social assistance, income replacement