ECON2915 2014 For the examiners

Problem 1 (50%)

Problem 1 is based on Lecture slides #6 (pp 26-29) as well as Weil chapter 8 (appendix).

a) Let $e \equiv A^{1/(1-\alpha)}$ and show that the production function can be rewritten as $Y = K^{\alpha}(eL)^{1-\alpha}$.

$$Y = AK^{\alpha}L^{1-\alpha} = e^{1-\alpha}K^{\alpha}L^{1-\alpha} = K^{\alpha}(eL)^{1-\alpha}$$

b) Let $y \equiv Y/(eL)$ and $k \equiv K/(eL)$ and show that the production function can be rewritten as $y = k^{\alpha}$ (the intensive form).

$$y = \frac{Y}{eL} = \frac{K^{\alpha}(eL)^{1-\alpha}}{eL} = \left(\frac{eL}{K}\right)^{-\alpha} = k^{\alpha}$$

c) Use the equations above to show that capital k accumulates according to

$$\dot{k} \quad = \quad \gamma k^{\alpha} - k(\delta + \hat{e}),$$

where $\hat{e}$ is the growth in e, i.e. $\hat{e} = \dot{e}/e$.

$$\dot{k} \quad = \quad \frac{d(K/eL)}{dt} = \frac{\dot{K}eL - \dot{e}LK}{(eL)^2} = \frac{\dot{K}}{eL} - \frac{\dot{e}}{e}k$$

Insert $\dot{K} = \gamma Y - \delta K$ and $y = k^{\alpha}$ to get $\dot{k} = \gamma k^{\alpha} - k(\delta + \hat{e})$.

d) Derive mathematically the steady state for capital

$$k^{*} = \left(\frac{\gamma}{\delta + \hat{e}}\right)^{1/(1-\alpha)}.$$

$\dot{k} = 0$ yields

$$\begin{aligned} \gamma k^{\alpha} &= k\left(\delta + \hat{e}\right) \\ k &= \left(\frac{\gamma}{\delta + \hat{e}}\right)^{1/(1-\alpha)} \end{aligned}$$

e) Consider a situation where the savings rate γ increases. Using a figure, explain what will happen to steady state k and y.

The steady state is found in the intersection between the γk^{α} amd $k\left(\delta + \hat{e}\right)$ curves (with k on the x-axis). A higher γ makes the γk^{α} shift up, which means that k^{*} must increase. Using the production function, we know that y^{*} also increases. The students should provide the intuition that higher savings rate means more investment and consequently more capital accumulation and output per effective capita $(Y/\left(eL\right))$.

f) Consider a situation where $\hat{e}$ increases.

(i) Using a figure, explain what will happen to k and y in the steady state.

The $k\left(\delta + \hat{e}\right)$ line becomes steeper, hence k^{*} and y^{*} goes down. k^{*} goes down because higher productivity growth means that the denominator in $K/\left(eL\right)$ grows more quickly (similar to the capital dilution effect of population growth).

(ii) Analyze what will happen to the steady state growth rate of GDP per capita (Y/L) and capital per capita (K/L). Hint: Use the fact that $\hat{y} = \hat{Y} - \left(\hat{e} + \hat{L}\right)$ (as above, the ˆ notation refers to growth rates).

Using $\hat{y} = \hat{Y} - \left(\hat{e} + \hat{L}\right)$, we get $\hat{Y} - \hat{L} = \hat{e}$ in the steady state (when $\hat{y} = 0$). The left hand side is growth in output per capita, which grows at the rate $\hat{e}$.

Using $\hat{k} = \hat{K} - \left(\hat{e} + \hat{L}\right)$, we get $\hat{K} - \hat{L} = \hat{e}$ in the steady state (when $\hat{k} = 0$). The left hand side is growth in capital per capita, which grows at the rate $\hat{e}$.

(iii) Is growth in Y/L higher or lower than growth in A?

By definition, $e \equiv A^{1/(1-\alpha)}$, so $\hat{e} = \frac{1}{1-\alpha}\hat{A}$. Hence $\widehat{(Y/L)} = \hat{e} > \hat{A}$. The candidate should provide the intuition that output per capita growth is determined by both productivity growth and capital deepening (K/L growth).

Problem 2 (20%)

(English) Growth decompositions tend to show that a major source of GDP growth comes from increases in A. Discuss briefly factors that determine the level and growth rate of A.

The candidate should provide a brief discussion of the role of technology (R&D, patenting, technology transfer) and efficiency (incentives, trade, competition, institutions, culture).

Problem 3 (30%)

(English) Consider a small open economy with two sectors (A and B) and two inputs (labor and capital). The prices of A and B are fixed on the world market. Sector A is relatively labor intensive, i.e. the ratio of labor to capital is higher in sector A than sector B. You do not need to make use of a full model to answer the questions, but you should justify your answers using figures where applicable.

Consider an increase in the supply of labor. Analyze the impact on output in sector A and B, when:

a) Capital is immobile and labor is mobile across sectors (the short run). Hint: You can use the equilibrium condition that the value of the marginal product of labor equals the wage.

The candidate should use the mixed specific factors model. E.g. using the diagram from lecture slides #10, p17, show that if labor supply L increases, then (i) the market clearing wage must be lower, and (ii) both sector A and B will employ more workers. With capital fixed and employment increasing in both sectors, output in both sector A and B must go up.

b) Both capital and labor are mobile across sectors (the long run).

The candidate should use the Hecksher-Ohlin model. By the Rybczynski theorem, an increase in the labor endowment leads to increased output in the labor intensive sector (sector A) and a decrease in the capital intensive sector (sector B). We can adapt the diagram from lecture slides #11, p15 as follows. Draw sector A output on the x-axis, sector B output on the y-axis. Draw two lines; the labor market clearing line (LC) and the capital market clearing line (KC). The slope of the LC curve is steeper (downward sloping) than the KC curve because of the assumed factor intensities. Higher labor endowment means that the LC curve shifts outward. Hence, the new intersection between LC and KC is down and to the right, with more output in sector A and less output in sector B.

c) Explain intuitively why the answer to b) (the long run) may be different than the answer to a) (the short run).

Higher labor supply means that in the short run, the return to capital will be higher in sector A (because they benefit more from lower wages). This will attract capital to sector A until the return is equal across sectors. Moving capital from B to A results in reduced output in sector B.