Problem 1:

a) Strict dominance:

A pure strategy $s_i$ of player $i$ is dominated if there is a strategy (pure or mixed) $\sigma_i \in \Delta S_i$ such that $u_i(\sigma_i, s_{-i}) > u_i(s_i, s_{-i})$ for all strategy profiles $s_{-i} \in S_{-i}$ of the other players.

Step 1: Dominance in pure strategies:

| | e | f |
|---|---|---|
| a | 2, - | 1, - |
| b | 1, - | 4, - |
| c | 2, - | 3, - |
| d | 3, - | 0, - |

1) No dominance in pure strategies.

2) Dominance with mixed strategies?

b and d combined can dominate a.
Pay off for strategy bd with weight $p$ on b gives pay off if player 2 chooses e of:

$p \cdot 1 + (1-p)\,3 = -2p + 3$

which is better than a if $-2p+3 > 2$
$p < \frac{1}{2}$.

and for f $\quad p\,4 + (1-p)\,0 > 1$
$p > \frac{1}{4}$

So combination of bd is better than a if weight on b is $\frac{1}{4} < p < \frac{1}{2}$.

Ex $p = \frac{1}{3}$

| | e | f |
|---|---|---|
| a | 2, - | 1, - |
| b d | $\frac{7}{3}$ | 4/3 |
| c | 3, - | 0, - |

$bd(e) = \frac{1}{3} \cdot 1 + \frac{2}{3} \cdot 3 = \frac{7}{3}$

$bd(f) = \frac{1}{3} \cdot 4 + \frac{2}{3} \cdot 0 = \frac{4}{3}$.

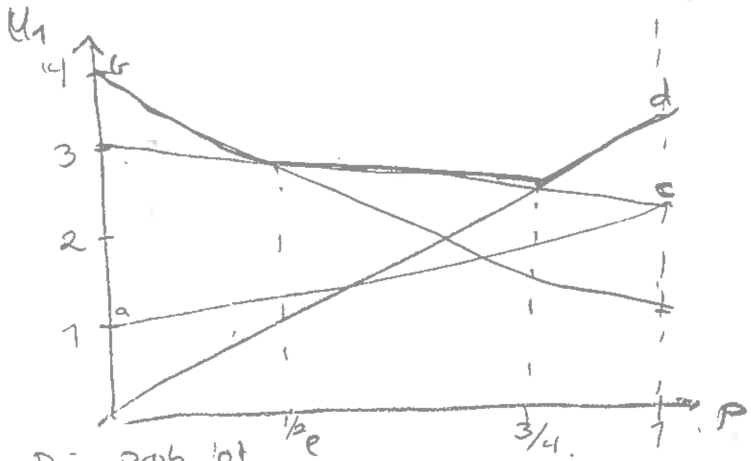
b) Best response:

Suppose player $i$ has a belief $\theta_{-i} \in \Delta S_i$ about the strategies played by the other players.

Player $i$'s strategy $s_i \in S_i$ is a best response if

$u_i(s_i, \theta_{-i}) \geq u_i(s'_i, \theta_{-i})$ for every $s'_i \in S_i$.

In a two player game BR can not be strictly dominated.




p = prob of

1-p = probability that player 2 plays f.

for low levels of p, d is the best response,

for medium levels of p - c is BR

for high levels of p b is best response.

a is never BR.

$E(c) > E(d)$ if $p \cdot 2 + (1-p) 3 > p \cdot 3 + (1-p) 0$

$2p + 3 - 3p > 3p$

$p < \frac{3}{4}$

$E(c) > E(b)$ if $p \cdot 2 + (1-p) \cdot 3 > p \cdot 1 + (1-p) \cdot 4$

$2p + 3 - 3p > p + 4 - 4p$

$2p > 1$

$p > 1/2$

Problem 2

Game 1

| | L | R |
|---|---|---|
| U | 0, 0 | 4, 2 |
| D | 2, 6 | 0, 8 |

Game 2

| | L | R |
|---|---|---|
| U' | 0, 2 | 0, 0 |
| D' | 2, 0 | 2, 2 |

a)

Bayesian normal form

Player 1 choose a strategy in each game, thus four possible strategies. Pay-off found by combining game 1 and 2 with equal prob

So for L: Player 1 , Player 2

UU' $\frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 0,\ \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 2$

UD' $\frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 2,\ \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 0$

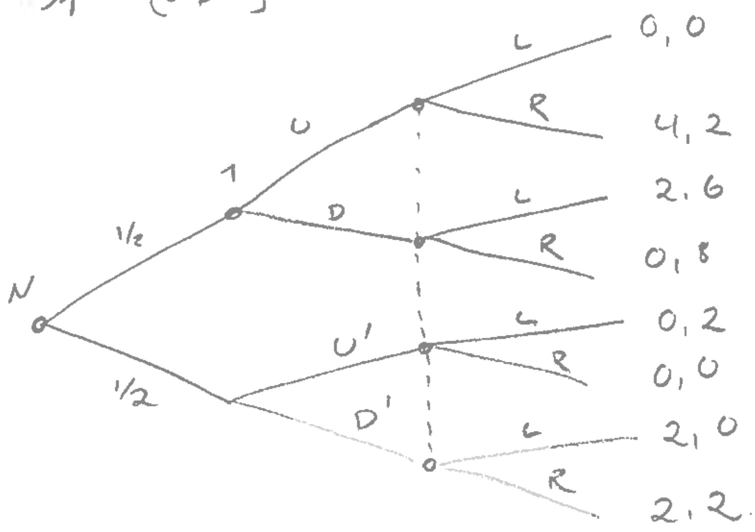
DU' $\frac{1}{2}\cdot 2 + \frac{1}{2}\cdot 0,\ \frac{1}{2}\cdot 6 + \frac{1}{2}\cdot 2$

DD' $\frac{1}{2}\cdot 2 + \frac{1}{2}\cdot 2,\ \frac{1}{2}\cdot 6 + \frac{1}{2}\cdot 0$

| | L | R |
|---|---|---|
| UU' | 0, 1 | 2, 1 |
| UD' | 1, 0 | 3, 2 |
| DU' | 1, 4 | 0, 4 |
| DD' | 2, 3 | 1, 5 |

b). One NE which is

$s_1 = \{UD'\}$ $s_2 = \{R\}$.



Preface - Seminar 9.

A mixed strategy profile is a MSNE if and only if:
for each player $i$:

* The expected payoff, given $\alpha^*_{-i}$, to every action to which $\alpha_i^*$ assigns positive probability is the same.

* The expected payoff, given $\alpha^*_{-i}$, to every action to which $\alpha_i^*$ assigns zero probability is at most the expected payoff to any other action to which $\alpha_i^*$ assigns positive probability.

i.e

1) The expected payoffs of the strategies played with positive probability must be the same, given the strategies of the other player.

2) The expected value of playing one of the strategies that are played with 0 probability can be no greater than the pay-off of those who are played.

Problem 3:

| | O | M |
|---|---|---|
| O | 3, 1 | 0, 0 |
| M | 0, 0 | 1, 3 |

| | O | M |
|---|---|---|
| O' | 0, 1 | 3, 0 |
| M' | 1, 0 | 0, 3 |

All strategies are rationalizable

a) Variant i has two pure equilibria

OO and MM

And one mixed equilibria where Player 1 is indifferent between mixing if player 2 plays O with probability p if p is given by:

$$p \cdot 3 + (1-p)\, 0 = 0 \cdot p + (1-p) \cdot 1$$
$$3p = 1 - p$$
$$p = \frac{1}{4}$$

And Player 2 is indifferent if player 1 mixes with probability q given by:

$$q \cdot 1 + (1-q) \cdot 0 = q \cdot 0 + (1-q) \cdot 3$$
$$q = 3 - 3q$$
$$q = \frac{3}{4}$$

So variant i has ~~three~~ equilibria:

$(O, O)$, $(M, M)$ and $\left(\left(\frac{3}{4}, \frac{1}{4}\right), \left(\frac{1}{4}, \frac{3}{4}\right)\right)$

Variant ii) No pure strategy eq.

Player 1 will mix if:

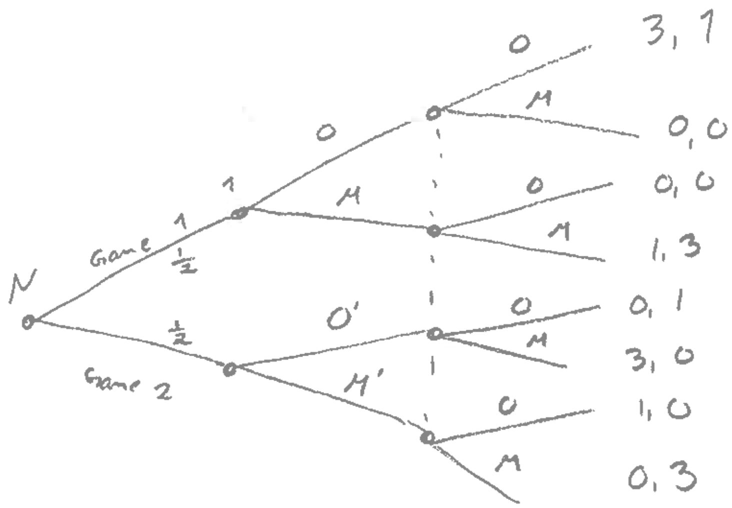
$$p \cdot 0 + (1-p) \cdot 3 = p \cdot 1 + (1-p) \cdot 0$$
$$3 - 3p = p \qquad p = \frac{3}{4}$$

Player 2 will mix if

$$q \cdot 1 + (1-q) \cdot 0 = q \cdot 0 + (1-q)\, 3$$
$$q = 3 - 3q \qquad \text{So eq: } \left(\left(\frac{3}{4}, \frac{1}{4}\right), \left(\frac{3}{4}, \frac{1}{4}\right)\right).$$
$$q = \frac{3}{4}$$

Problem 3:

a)




OO', O for player 1:

$0{,}5 \cdot 3 + 0{,}5 \cdot 0 = 1.5.$

Player 2:

$0{,}5 \cdot 1 + 0{,}5 \cdot 1 = 1.$

OO'M 1:

$0{,}5 \cdot 0 + 0{,}5 \cdot 3 = 1.5.$

2: $0.5 \cdot 0 + 0{,}5 \cdot 0 = 0.$

Problem 3G

| | O | M |
|---|---|---|
| OO' | 1.5, 1 | 1.5, 0 |
| OM' | 2, 0.5 | 6. 1.5 |
| MO' | 0, 0.5 | 2, 1.5 |
| ~~MM'~~ | ~~0.5, 0~~ | ~~0.5, 3~~ |

3C)

Proposition:

A mixed strategy profile $\alpha^*$ in a strategic game with vNM preferences in which each player has finitely many actions is a mixed strategy NE if and only if:

1) The expected payoff, given $\alpha^*_{-i}$, to every action in which $\alpha^*_i$ assigns positive probability is the same.

2) The expected payoff, given $\alpha^*_{-i}$, to every action to which $\alpha^*_i$ assigns zero probability is at most the expected payoff to any action to which $\alpha^*_i$ assigns positive probability.

This gives three possible types of NE

a) Pure × Pure, both play pure

b) Pure × mixed, one play pure and one mix

c) Mixed × mixed, both mixes.

i) So there is one NE of type a.

$S_1 = \{MO'\}$ $S_2 = \{M\}$.

ii) There are no NE for type b in this game. If player 1 plays a pure strategy player 2 do not want to mix since L or R gives strictly higher pay off than mixing given the pure strategy of player 1. And vice versa.

iii) Player 1 can mix in multiple ways.

1) Mix only strategy OO' and OM'
2) Mix only strategy OO' and MO'
3) Mix only strategy OM' and MO'
4) Mix OO', OM' and MO'

Mixing with MM' can never be an equilibrium since MM' is a strictly dominated strategy.

Strategy 1:

According to the proposition strategy 1 is a NE if

$$E(OO') = E(OM') \geq E(MO')$$

where p is player 1's belief about player 2's probability of playing O.

$$p \cdot 1.5 + (1-p) \cdot 1.5 = p \cdot 2 + (1-p) \cdot 0 \geq p \cdot 0 + (1-p) \cdot 2$$

$$1.5 = 2p \geq 2 - 2p$$

First part gives $p = \frac{3}{4}$ and given $p = \frac{3}{4}$

$$2 \cdot \frac{3}{4} > 2 - 2 \cdot \frac{3}{4}$$

$$\frac{6}{4} > \frac{2}{4}.$$

So player 1 will be willing to randomize between $OO'$ and $OM'$ if player 2 plays $O$ with probability $\frac{3}{4}$.

Will player 2 randomize?
He will if $E(O) = E(M)$. where $q$ is weight he attaches on belief that player 1 plays $OO$ and $1-q$ on playing $OM'$.
Which gives

$$q \cdot 1 + (1-q) \cdot 0.5 = q \cdot 0 + (1-q) \cdot 1.5$$
$$q + 0.5 - 0.5q = 0 + 1.5 - 1.5q$$
$$2q = 1$$
$$q = \frac{1}{2}.$$

So one NE where

$$S_1 = \{ \tfrac{1}{2}, \tfrac{1}{2}, 0, 0 \} \qquad S_2 = \{ \tfrac{3}{4}, \tfrac{1}{4} \}.$$

<u>Strategy 2:</u>
Requirement: $E(OO') = E(MO') \geq E(OM')$.

$$p \cdot 1.5 + (1-p) \cdot 1.5 = p \cdot 0 + (1-p) \cdot 2 \geq p \cdot 2 + (1-p) \cdot 0$$
$$1.5 = 2 - 2p \geq 2p$$

First part gives $p = \frac{1}{4}$.

Will player 2 randomize? Again he will randomize if $E(O) = E(M)$ let $r$ be weight he puts on probability that player 2 plays $OO$ and $(1-r)$ on $MO'$. Then

$$r \cdot 1 + (1-r) \cdot 0.5 = r \cdot 0 + (1-r) \cdot 1.5$$
$$r = \frac{1}{2}.$$

So NE where

$$S_1 = \{\tfrac{1}{2}, 0, \tfrac{1}{2}, 0\} \qquad S_2 = \{\tfrac{1}{4}, \tfrac{3}{4}\}.$$

Is strategy 3 a NE?

Player 1 is willing to randomize between OM' and MO' if: $E(OM') = E(MO') \geq E(OO')$

$$p \cdot 2 + (1-p) \cdot 0 = 0 \cdot p + (1-p) 2 \geq p \cdot 1.5 + (1-p) \cdot 1.5$$

First part gives $2p = 2 - 2p$

$$p = \tfrac{1}{2}.$$

But with $p = \frac{1}{2}$ the inequality doesn't hold as

$$2 \cdot \tfrac{1}{2} < 1.5.$$

Thus strategy 3 is not a NE.

Is strategy 4 a NE?

No. Because for that to be the case

$$E(OM') = E(MO') = E(OO')$$

$$p \cdot 2 + (1-p) \cdot 0 = 0 \cdot p + (1-p) \cdot 2 = p \cdot 1.5 + (1-p) \cdot 1.5$$

$$2p = 2 - 2p = 1.5.$$

No $p$ is possible to make this equality hold.

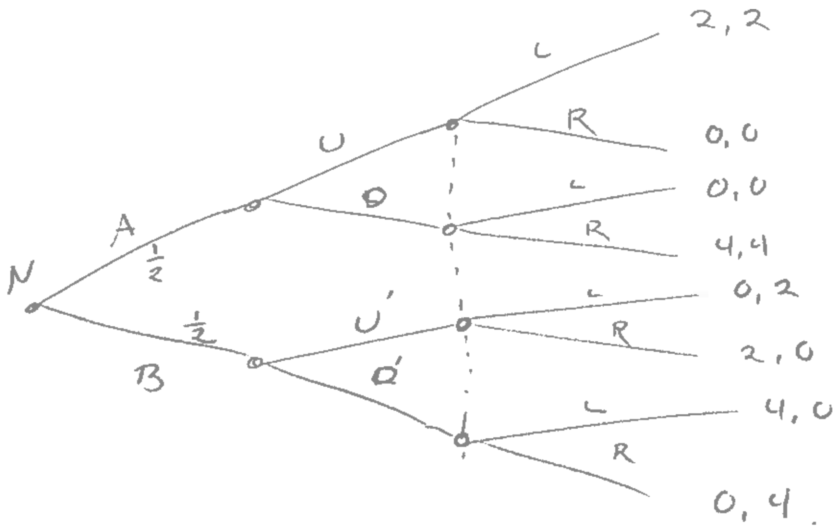
Problem 4:

W 26.3:

| | L | R |
|---|---|---|
| U | 2,2 | 0,0 |
| D | 0,0 | 4,4 |

| | L | R |
|---|---|---|
| U' | 0,2 | 2,0 |
| D' | 4,0 | 0,4. |

a)




Player 1:

UU'L : $0,5 \cdot 2 + 0,5 \cdot 0 = 1$

UU'R $0,5 \cdot 0 + 0,5 \cdot 2 = 1$

UD'L : $0,5 \cdot 2 + 0,5 \cdot 4 = 3$

UD'R $0,5 \cdot 0 + 0,5 \cdot 0 = 0$

Player 2:

$0,5 \cdot 2 + 0,5 \cdot 2 = 2$

$0,5 \cdot 0 + 0,5 \cdot 0 = 0$

$0,5 \cdot 2 + 0,5 \cdot 0 = 1$

$0,5 \cdot 0 + 0,5 \cdot 4 = 2$

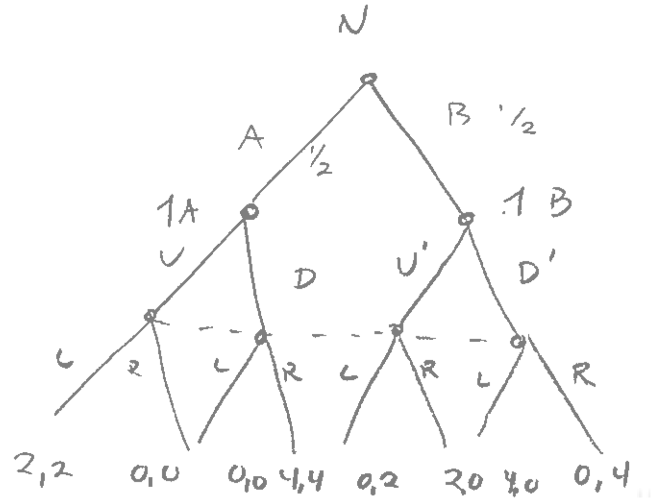
| | L | R |
|---|---|---|
| ~~UU'~~ | ~~1,2~~ | ~~1,0~~ |
| UD' | ~~3,1~~ | ~~0,2~~ |
| DU' | ~~0,1~~ | 3,2 |
| DD' | ~~2,0~~ | ~~2,4~~ |

UU' dom by DD'

L dom by R

UD' & DD' dom by DU'

b)




1A: U

| 1B | L | R |
|---|---|---|
| U' | 2, 0, 2 | 0, 2, 0 |
| D' | 2, 4, 1 | 0, 0, 2 |

1A: D

| 1B | L | R |
|---|---|---|
| U' | 0, 0, 1 | 4, 2, 2 |
| D' | 0, 4, 0 | 4, 0, 4 |

Player 2:

UU'L $0{,}5 \cdot 2 + 0{,}5 \times 2 = 2$

UU'R $0{,}5 \cdot 0 + 0{,}5 \times 0 = 0$

UD'L $0{,}5 \times 2 + 0{,}5 \times 0 = 1$

UD'R $0{,}5 \times 0 + 0{,}5 \cdot 4 = 2$

One NE

Rationalizability:
Each player has only two strategies. Neither are strictly dominated. With only two strategies you cannot combine to dominate either.

c) Player 1A and 1B's belief in the first term must coincide since it is the same player and they haven't observed the nature yet.

In game 2 the players 1A & 1B may have different beliefs about player 2's strategies when applying rationalizability.

In eq beliefs must coincide.

26.5. <u>Problem 5:</u>

- Demand: $q_1 = 22 - 2p_1 + p_2$

$q_2 = 22 - 2p_2 + p_1$

- $c(q_1) = 10q_1$

$c(q_2) = c\,q_2.$

a) Write the firms pay-off functions:

$\Pi_1 = p_1 q_1 - 10q_1 = q_1(p_1 - 10)$

$= (22 - 2p_1 + p_2)(p_1 - 10)$

$\Pi_2 = p_2 q_2 - c q_2 = q_2(p_2 - c)$

$= (22 - 2p_2 + p_1)(p_2 - c).$

b) Calculate Best response functions:

Firm 1: $\frac{\partial \Pi_1}{\partial p_1} = -2(p_1 - 10) + 22 - 2p_1 + p_2 = 0$

$4p_1 = 42 + p_2$

$BR_1(p_2):\ p_1 = \frac{42 + p_2}{4}$

Firm 2: $\frac{\partial \Pi_2}{\partial p_2} = -2(p_2 - c) + 22 - 2p_2 + p_1 = 0$

$4p_2 = 22 + p_1 + 2c$

$BR_2(p_1):\ p_2 = \frac{22 + 2c + p_1}{4}$

c) From G we have that

1) $p_1 = \frac{42 + p_2}{4} = \frac{42}{4} + \frac{1}{4} p_2.$

and 2) $p_2 = \frac{1}{4} ( 22 + 2c + p_1 )$

Thus enter 2) into 1). thus:

$$p_1 = \frac{42}{4} + \frac{1}{4} \cdot \frac{1}{4} (22 + 2c + p_1) \quad \Big| \cdot 16$$

And enter that $c = 10$ gives:

$$16 p_1 = 42 \times 4 + 22 + 2 \times 10 + p_1$$

$$15 p_1 = 42 \times 5$$

$$p_1 = \frac{42 \cdot \cancel{5}}{\cancel{15}} = \frac{42}{3} = 14$$

and $p_2 = \frac{42 + p_1}{4} = \frac{42 + 14}{4} = 14$

d) Assume two types of player 2 one with Low costs ($c = 6$) and one with high ($c = 14$). each with equal probability.

$$3)\quad BR_2^L(p_1) = \frac{22 + 2 \cdot 6 + p_1}{4} = \frac{34 + p_1}{4}.$$

i.e best response function of firm 2 with low costs.

$$4)\quad BR_2^H(p_1) = \frac{22 + 2 \times 14 + p_1}{4} = \frac{50 + p_1}{4}.$$

Now firm 1 has a new profit function:

$$1^*)\quad \pi_1 = \left(22 - 2p_1 + \frac{1}{2} p_2^H + \frac{1}{2} p_2^L\right)(p_1 - 10).$$

i.e. with ½ probability he meets firm 2 which have high costs and ½ probability he meets low costs.

Thus

$$5)\quad BR_1(p_2) = \frac{42}{4} + \frac{1}{4}\left(\frac{1}{2} p_2^H + \frac{1}{2} p_2^L\right)$$

Enter 3) and 4) into 5). gives:

$$p_1 = \frac{42}{4} + \frac{1}{8}\left(\frac{34 + p_1}{4} + \frac{50 + p_1}{4}\right) \quad \Big| \cdot 32$$

$$32 p_1 = 8 \times 42 + 34 + p_1 + 50 + p_1$$

$$30\, p_1 = 8 \times 42 + 84 = 10 \times 42$$

$$p_1 = \frac{\overset{1}{\cancel{10}} \times 42}{\cancel{30}\ 3} = 14$$

$$p_2^L = \frac{34 + 14}{4} = 12 \qquad p_2^H = \frac{50 + 14}{4} = 16.$$