Microeconomics 3200/4200:

Part 1

P. Piacquadio

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Outline

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Course outline: part 1

- Part 1 is about microeconomics:
  - short introduction (Lecture 1);
  - consumer theory (Lectures 1-4);
  - partial equilibrium (Lecture 5);
  - production theory (Lectures 6-8);
  - uncertainty (Lecture 9).

Course outline: part 2

- Part 2 is about game theory (Lectures 10-16);
- with Professor Geir Asheim.

Seminars

- Seminar teachers are:
  - Torje Hegna (torje.hegna@econ.uio.no); and
  - Seongbong Hong (seongbong.hong@econ.uio.no).

Exam

- (Final) written examination on December 15th at 14:30 (3 hours).
- Compulsory assignment will be available in FRONTER:
  - 2 tests:
    - "micro," 3 Oct at 9:00 to Oct 5 at 15:00; and
    - "game theory," 7 Nov at 9:00 to 9 Nov at 15:00.
  - each test consists of 10 multiple-choice questions;
  - to pass the compulsory assignment you must:
    - submit answers to both tests;
    - answer correctly 11/(10+10) questions.
- For more information, see course page.

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Economic models

- Economics is about almost everything:
  - Varian writes: "economics proceeds by developing models of social phenomena."
- Why models?
- Two basic principles:
  - optimization principle;
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  - optimization principle;
  - equilibrium principle.

The optimization principle

- *People try to do what is best for them, given the available alternatives.*

- This is quite reasonable.
- The assumption tells that if Andrea decides to spend her savings on a new bike, it must be true that it is in her best interest to do so...

- ...given her information about the available alternatives, given her quantity of saving, given the prices of commodities, given what her friends decided to do, etc.

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- *Prices adjust and ensure that the "demand" meets its "supply."*

- This is somewhat more demanding.
- Sometimes prices adjust too slowly or too much. Sometimes other things happen before reaching the equilibrium, so that differences in demand and supply may increase.
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An example: the rental market

- Let us look at the rental market around Blindern.

- We can start investigating the **demand side**:
  - How many students are willing to pay 15.000 NOK?
  - How many are willing to pay 14.000 NOK?
  - How many are willing to pay 13.000 NOK?
  - ...

- The **reservation price** is the largest price that each student would be willing to pay.
- This information can be summarized compactly in a graph.

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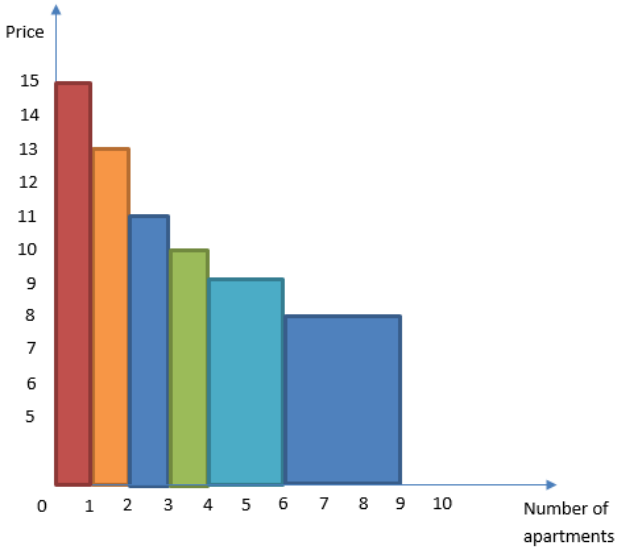
Illustration: demand curve



Few more assumptions

- To simplify and avoid jumps, we assume that:
  - there are many students looking to rent;
  - units are homogeneous (say 1-bedroom apartments).

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Supply side

- In the short run, the number of apartments for rent is fixed.
- Assume that:
  - all students are equal: landlords only care about the rent price;
  - rental market is flexible: if a new student comes and proposes a larger rent, the landlord can reassign the apartment;
  - thus, all landlords will rent at the same price.
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Rental market equilibrium

- The equilibrium is defined by:
  - the equilibrium number of apartments rented x^*; and
  - the equilibrium price p^*.

- Why equilibrium?
  - If price was $p > p^*$, then less apartment would be rented.
  - The landlords with empty apartments would be willing to rent at a lower price $p' < p$.

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Comparative statics

- Increase in supply:
  - the supply curve shiftes to the right;
  - more apartments are available;
  - equilibrium price decreases.
- Some students jointly rent a house:
  - the demand curve shiftes to the left; and
  - fewer students are willing to rent 1-bedroom apartments;
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- Tax on rentals?

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Outline

Consumption bundle

- There are two goods, good 1 and good 2.
- Andrea's **consumption bundle** is denoted $\mathbf{x} \equiv (x_1, x_2)$.
  - (x_1, x_2) is a vector, i.e. an ordered list of numbers where x_1 is the quantity of good 1 and x_2 is the quantity of good 2.
  - (for simplicity) each number is a non-negative real number;
  - goods are perfectly divisible and privatly appropriable;
  - the consumption space is $X \equiv \mathbb{R}^n_+$ with $n = 2$;
  - for example, you can think of good 1 as milk and good 2 as a composite good representing everything else Andrea might want to purchase.

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Consumption prices

- Each good has a price. Let $\boldsymbol{p} \equiv (p_1, p_2)$ be the **price vector**.
  - (p_1, p_2) is another vector: p_1 is the price of good 1 and p_2 is the price of good 2.

- What is $p_1 x_1$? it is the money Andrea spends to purchase x_1 quantity of good 1 at price p_1.
- Similarly, $p_2 x_2$ is the money Andrea spends to purchase x_2 quantity of good 2 at price p_2.

- We say that (x_1, x_2) is **affordable** for Andrea if he has enough money m to purchase such bundle, that is, if:

$$p_1 x_1 + p_2 x_2 \leq m.$$

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Budget set

- Then, the **budget set** is the set of all consumption bundles that Andrea can afford at prices (p_1, p_2) and income m. All (x_1, x_2) such that
$$p_1x_1 + p_2x_2 \leq m.$$
- The **budget line** is the frontier of the budget set. It is the set of all consumption bundles that Andrea can (exactly) buy when spending all her money m. All (x_1, x_2) such that
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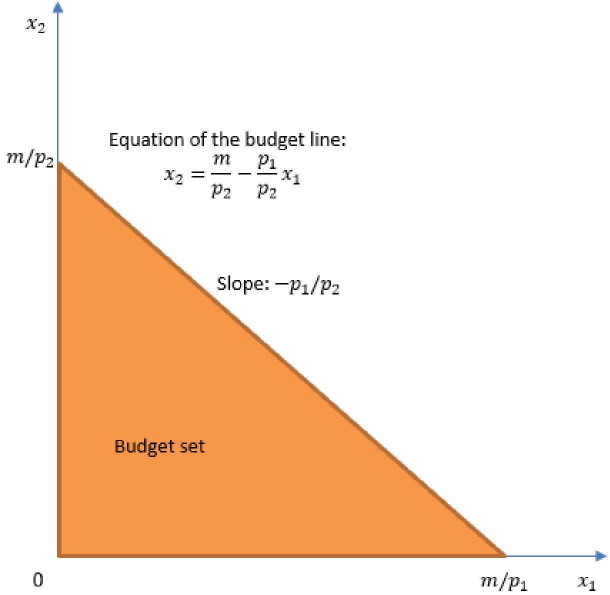
Budget set: illustration



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- What happens when m increases?
- What happens when p_1 decreases?
- What happens with inflaction?
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Consumers' preferences

- When Andrea faces her budget set, she has a choice to make: which consumption bundle to choose?

- Building on the optimizing principle, the answer is: the bundle she prefers better.

- **Preferences**, denoted $\succsim$, capture this information:
  - we write $(x_1, x_2) \succsim (x_1', x_2')$ if Andrea finds the consumption bundle (x_1, x_2) at least as desirable as the consumption bundle (x_1', x_2');
  - we write $(x_1, x_2) \succ (x_1', x_2')$ if Andrea prefers (x_1, x_2) to (x_1', x_2');
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Relation between preference symbols

- If $(x_1, x_2) \succsim (x_1', x_2')$, but not $(x_1, x_2) \sim (x_1', x_2')$, then $(x_1, x_2) \succ (x_1', x_2')$.
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Assumptions about preferences

- Why imposing assumptions on preferences?
  - eliminate unreasonable cases (i.e. $(x_1, x_2) \succ (x_1', x_2')$ and $(x_1', x_2') \succ (x_1, x_2)$);
  - obtain more far reaching results.

- **Complete.** For each pair of consumption bundles $(x_1, x_2), (x_1', x_2') \in X$, either
  - $(x_1, x_2) \succsim (x_1', x_2')$; or
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Assumptions about preferences

- **Reflexive.** For each consumption bundle $(x_1, x_2) \in X$,
  - $(x_1, x_2) \succsim (x_1, x_2)$.
- **Transitive.** For each triplet of consumption bundles $(x_1, x_2), (x_1', x_2'), (x_1'', x_2'') \in X$,
  - $(x_1, x_2) \succsim (x_1', x_2')$ and $(x_1', x_2') \succsim (x_1'', x_2'')$ implies that $(x_1, x_2) \succsim (x_1'', x_2'')$.

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More on preferences

- Consider the consumption bundle $(x_1, x_2) \in X$. The **indifference curve** through (x_1, x_2) is the set of all consumption bundles $(x_1', x_2') \in X$ such that $(x_1', x_2') \sim (x_1, x_2)$.
  - Indifference curves cannot cross;
  - preferences consist of all indifference curves.

- The **upper-contour set** at (x_1, x_2) is the set of all consumption bundles $(x_1', x_2') \in X$ such that $(x_1', x_2') \succsim (x_1, x_2)$.

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- Goods, bads, and neutral goods
- Satiation
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Convexity

- Perfect substitutes

- Perfect complements

- **Convexity.** For each pair of consumption bundles $(x_1, x_2), (x_1', x_2') \in X$ with $(x_1, x_2) \sim (x_1', x_2')$ and each $t \in [0, 1]$,

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- **Convexity.** For each pair of consumption bundles $(x_1, x_2), (x_1', x_2') \in X$ with $(x_1, x_2) \sim (x_1', x_2')$ and each $t \in [0, 1]$,

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Marginal rate of substitution

- The marginal rate of substitution (MRS) expresses the rate at which a consumer, Andrea, is just willing to substitute a good for another one.
- This is a local concept!
- The MRS at (x_1, x_2) is the slope at (x_1, x_2) of the indifference curve through (x_1, x_2).
- When is the MRS well-defined?
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The concept

- Utility is a problematic name: its meaning changed over time and is still a big source of confusion.
  - some economists consider it as a measure of happiness or subjective well-being;
  - others take it as a different way to express the same information of preferences.

- We shall go with the second interpretation. A **utility function** U is a numerical representation of preferences $\succsim$. Then, for each pair $(x_1, x_2), (x_1', x_2') \in X$:

$$(x_1, x_2) \succsim (x_1', x_2')$$

IF and ONLY IF

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Existence and uniqueness of a utility function

Theorem

If preferences $\succsim$ are complete, transitive, and continuous, then there exists a continuous utility function U that represents $\succsim$.

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Examples of utility functions

- $U(x_1, x_2) = x_1 x_2$;
- $U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$ with $\alpha \in [0, 1]$;

- $U(x_1, x_2) = ax_1 + bx_2$ with $a, b > 0$;
- $U(x_1, x_2) = \min[ax_1, bx_2]$ with $a, b > 0$;

- $U(x_1, x_2) = x_1 + v(x_2)$ with v and increasing function;
- $U(x_1, x_2) = \left[a(x_1)^{\rho} + (1-a)x_2^{\rho}\right]^{\frac{1}{\rho}}$ with $a \in [0, 1]$ and $\rho > 0$.

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Marginal utility (See appendix Ch.4)

- If the function U is differentiable, then the derivative U wrt the quantity of the good gives the **marginal utility**.
- If x_1 increases to $x_1 + \Delta x_1$, the utility goes from u to $u + \Delta u$. Then,

$$u + \Delta u = U(x_1 + \Delta x_1, x_2)$$

- or, since $u = U(x_1, x_2)$,

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- Now, divide both sides by Δx_1:

$$\frac{\Delta u}{\Delta x_1} = \frac{U(x_1 + \Delta x_1, x_2) - U(x_1, x_2)}{\Delta x_1}$$

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What is the optimal choice of the consumer?

- There are two ways to look at the problem.
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The utility maximization problem

- What is the consumption bundle (x_1^*, x_2^*) that maximizes the utility of Andrea, given prices (p_1, p_2) and money m?

- The problem can be written as follows:

$$\begin{array}{rl} \max_{(x_1,x_2)\in X} & U(x_1, x_2) \\ s.t. & p_1 x_1 + p_2 x_2 \leq m \end{array} \tag{1}$$

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The algebraic solution

- First, write the Lagrangian of the maximization problem:

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- The FOCs require that there exists $\lambda^* \geq 0$ such that:

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Marshallian demand function

- The solution of the utility maximization problem is one (or more) consumption bundles. These depend on the prices (p_1, p_2) and money m.

- Assume there is a single optimum. Let $D^1(p_1, p_2, m)$ be the function that tells the optimal amount of good 1 for each prices and money. Let $D^2(p_1, p_2, m)$ be the function that tells the optimal amount of good 2 for each prices and money. These are the **Marshallian** (or ordinary or Walrasian or uncompensated) **demand functions**.

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Properties of the Marshallian demand functions

- The few assumptions we made on preferences are far reaching. *Completeness*, *transitivity*, *continuity*, and *strict convexity* together imply that the Marshallian demand functions:

- are continuous in prices and money;
- are homogeneous of degree 0 with respect to prices and money;
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The indirect utility function

- Substituting the the solution of the utility maximization problem into the utility function gives the **indirect utility function**.

- It answers the following question: what is the maximum amount of utility one can reach by choosing optimally the consumption bundle with prices (p_1, p_2) and money m?

- It is thus a function of prices (p_1, p_2) and money m:

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- $U(x_1, x_2) = x_1 x_2$.
- Be smart! Use the equivalent (why?) utility function $\bar{U}(x_1, x_2) = \ln x_1 + \ln x_2$.
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- Substitute in the budget line and solve:

$$m = p_1 x_1^* + p_2 \cdot \frac{p_1}{p_2} x_1^*$$

$$D^1(p_1, p_2, m) = x_1^* = \frac{1}{2} \frac{m}{p_1}$$

- Since $\frac{1}{2}m$ is optimally spent for x_1^*, the other half is spent for x_2^*:
-

$$D^2(p_1, p_2, m) = x_2^* = \frac{1}{2} \frac{m}{p_2}.$$

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More examples

- $U(x_1, x_2) = ax_1 + bx_2$.
- $U(x_1, x_2) = \min[ax_1, bx_2]$.

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Outline

The cost minimization problem

- What is the consumption bundle (x_1^*, x_2^*) that minimizes the money spent by Andrea, given prices (p_1, p_2) and a goal level of utility $u \leq U(x_1^*, x_2^*)$?

- The problem can be written as follows:

$$\begin{aligned} \min_{(x_1,x_2)\in X} \quad & p_1 x_1 + p_2 x_2 \\ s.t. \quad & u \leq U(x_1^*, x_2^*) \end{aligned} \tag{6}$$

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The algebraic solution

- First, write the Lagrangian of the minimization problem:

$$\mathscr{L}(\mathbf{x}, \lambda; \mathbf{p}, u) = p_1 x_1 + p_2 x_2 + \lambda [u - U(x_1, x_2)] \quad (7)$$

- The FOCs require that there exists $\lambda^* \geq 0$ such that:

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- Then, if U satisfies monotonicity and $p_1, p_2 > 0$, then

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- If $x_1^*, x_2^* > 0$, then $p_1 = \lambda^* MU_1(x_1^*, x_2^*)$ and $p_2 = \lambda^* MU_2(x_1^*, x_2^*)$. Thus:

$$\frac{MU_1(x_1^*, x_2^*)}{MU_2(x_1^*, x_2^*)} = \frac{p_1}{p_2} \tag{10}$$

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Outline

Hicksian demand function

- The solution of the cost minimization problem is one (or more) consumption bundles. These depend on the prices (p_1, p_2) and the utility level u.

- Assume there is a single optimum. Let $H^1(p_1, p_2, u)$ be the function that tells the optimal amount of good 1 for each prices and utility level. Let $H^2(p_1, p_2, u)$ be the function that tells the optimal amount of good 2 for each prices and utility level. These are the **Hicksian** (or compensated) **demand functions**.

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Properties of the Hicksian demand functions

- *Completeness, transitivity, continuity,* and *strict convexity* together imply that the Hicksian demand functions:
  - are continuous in prices and utility;
  - are homogeneous of degree 1 with respect to prices;
  - satisfy: $U\left(H^1(p_1, p_2, u), H^2(p_1, p_2, u)\right) = u$.

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The cost function

- The **cost function** is the cost of the consumption bundle(s) that solves the cost minimization problem.

- It answers the following question: what is the amount of money needed to choose optimally a consumption bundle that achieves the utility level u at prices (p_1, p_2)?

- Thus, the cost function depends on the prices (p_1, p_2) and the utility level u:

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Outline

Example 1

- $U(x_1, x_2) = x_1 x_2$.
- You can use again the equivalent function $\bar{U}(x_1, x_2) = \ln x_1 + \ln x_2$. But:

$$u \leq U(x_1, x_2) \Leftrightarrow \ln u \leq \ln x_1 + \ln x_2$$

- $\mathscr{L}(\mathbf{x}, \lambda; \mathbf{p}, u) = p_1 x_1 + p_2 x_2 + \lambda [\ln u - \ln x_1 - \ln x_2]$;
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- By monotonicity of $\bar{U}$ and $p_1, p_2 > 0$: $\ln u = \ln x_1^* + \ln x_2^*$.
- Since $x_1^*, x_2^* > 0$ (why?),

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Outline

Identities

- How are demands, indirect utility, and cost function related?
  - $H^i(p_1, p_2, u) = D^i(p_1, p_2, C(p_1, p_2, u))$;
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Outline

Income effects

- Having a solution for each prices and each level of money allows us to study what happens when changing these parameters of the decision problem.

- We start with a change of the money level: how does the Marshallian demand change when money changes?
  - if $\frac{\partial D^i}{\partial m} \geq 0$, then i is a **normal good**.
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Price effects

- What happens on the demand of good i when the price of good j changes?
- Let us start from our previous observation that:

$$H^{i}(\mathbf{p},u)=D^{i}(\mathbf{p},C(\mathbf{p},u))$$

- Take the derivative w.r.t. p_j:

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Price effects...

- But since $x_j^* = \frac{\partial C(\mathbf{p},v)}{\partial p_j} = C_j(\mathbf{p},u)$, we get the **Slutsky equation**:

$$D_j^i(\mathbf{p},m) = H_j^i(\mathbf{p},v) - x_j^* D_m^i(\mathbf{p},m)$$

- The total effect of a price change $D_j^i(\mathbf{p},m)$ is the sum of a substitution effect $H_j^i(\mathbf{p},v)$ and an income effect $-x_j^* D_m^i(\mathbf{p},m)$.

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- But since $x_j^* = \frac{\partial C(\mathbf{p},v)}{\partial p_j} = C_j(\mathbf{p},u)$, we get the **Slutsky equation**:

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- The total effect of a price change $D_j^i(\mathbf{p},m)$ is the sum of a substitution effect $H_j^i(\mathbf{p},v)$ and an income effect $-x_j^* D_m^i(\mathbf{p},m)$.

Income effects

- To repeat, if $D^i_m(\mathbf{p}, m)$ is negative, the ordinary demand for good i is decreasing with income: then i is an **inferior good**.

- If $D^i_m(\mathbf{p}, m)$ is non-negative, the ordinary demand for good i is non-decreasing with income: then i is a **normal good**.

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Substitution effects

- $H_j^i(\mathbf{p}, v) = \frac{\partial \frac{\partial C(\mathbf{p},v)}{\partial p_i}}{\partial p_j} \equiv C_{ij}(\mathbf{p}, v) = C_{ji}(\mathbf{p}, v) \equiv \frac{\partial \frac{\partial C(\mathbf{p},v)}{\partial p_j}}{\partial p_i} = H_i^j(\mathbf{p}, v)$

- Thus: the substitution effects are symmetric!

- If $H_j^i(\mathbf{p}, v) > 0$, goods i and j are **net substitutes**: an increase in price of good j **increases** the Hicksian demand for good i.
- If $H_j^i(\mathbf{p}, v) < 0$, goods i and j are **net complements**: an increase in price of good j **decreases** the Hicksian demand for good i.

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The own price effect

- We can look at the effect of a variation of the price of good i on the demand of good i:

$$D_i^i(\mathbf{p}, m) = H_i^i(\mathbf{p}, v) - x_i^* D_m^i(\mathbf{p}, m)$$

- By the concavity of the cost function $H_i^i(\mathbf{p}, v) = C_{ii}(\mathbf{p}, v) < 0$.
- What about the income effect? the income effect can be both positive or negative.
- If both $D_m^i(\mathbf{p}, m) < 0$ (inferior good) and $x_i^* D_m^i(\mathbf{p}, m) < H_i^i(\mathbf{p}, v) < 0$, then the negative income effect dominates the substitution effect and the total effect is positive, i.e. $D_i^i(\mathbf{p}, m) > 0$: increasing the price of good i increases the demand of good i. Then i is a **Giffen good**.
- If i is a normal good, $D_i^i(\mathbf{p}, m) < 0$: demand decreases when the price increases.

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Outline

The consumer's surplus

- Assume price of good 1 decreases from $\bar{p}_1$ to p_1'. How to measure the benefit of the price change on a consumer?

- graphically.

- Algebraically. Using the **consumer's surplus** CS.
- The consumer's surplus at prices $\bar{p}_1$ is:

$$CS(\bar{p}_1, p_2, m) = \int_{\bar{p}_1}^{\infty} D^i(p_1, p_2, m)\, dp_1.$$

- Thus, the welfare gain for the price reduction is:

$$\begin{aligned}\Delta CS &= CS(\bar{p}_1, p_2, m) - CS(p_1', p_2, m)\\ &= \int_{p_1'}^{\bar{p}_1} D^1(p_1, p_2, m)\, dp_1.\end{aligned}$$

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Outline

2 further measures: compensated variation

- What is the money change that would make the consumer indifferent between the "initial" consumption $D^1(\bar{p}_1, p_2, m), D^2(\bar{p}_1, p_2, m)$ and a consumption bundle at prices p_1'?

- the **compensated variation** CV is such that

$$v = V\left(p_1', p_2, m - CV\right);$$

  - or

$$CV\left(\bar{p}_1 \to p_1'\right) = C\left(\bar{p}_1, p_2, v\right) - C\left(p_1', p_2, v\right).$$

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2 further measures: equivalent variation

- What is the money change that would make the consumer indifferent between the "final" consumption $D^1(p_1', p_2, m), (p_1', p_2, m)$ and a consumption bundle at prices $\bar{p}_1$?

- the **equivalent variation** EV is such that

$$v' = V(\bar{p}_1, p_2, m + EV);$$

  - or

$$EV(\mathbf{p} \to \mathbf{p}') = C(\bar{p}_1, p_2, v') - C(p_1', p_2, v').$$

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