Basic model of impact of forest fires and dryness on forest value

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The model

A purpose of this analysis is to present an analysis of what is behind the concept of “marginal rainforest value”. In doing that I also provide a basis for forest fire occurrence as a separate factor behind the marginal valuation of rainforests.

We use a simple approach for defining the value of a (small piece of) rainforest, as follows:

\[ rV = (1 - L)[v(D) + w(D)] + \lambda(L, D)(V_F - V) + (g - \theta)V. \]

The main variables that enter into this analysis are defined as follows.

- \( rV \) = current returns to the capitalized asset value of a given small part of the rainforest.
- \( 1-L \) = currently remaining forest on a unit of land area that was, initially, fully forested.
- \( D \) = forest dryness, which is a function of the share of lost forest, \( D(L) \).
- \( r \) = periodic interest rate for discounting of future costs and benefits, assumed constant.
- \( \lambda \) = frequency (continuous-time probability) of fire occurrence, where \( \lambda = \lambda(L, D) \) with \( \lambda'(L, D) > 0, \lambda'(D, L) > 0 \) (less forest on a given plot, and more generally drier forest, both raise the probability of future fire for the remaining forest on that plot).
- Note that dryness \( D \) is a biome-wide phenomenon implying that the \( \lambda \) function, considering impacts on \( D \), is a biome-wide average and not a plot-specific value.
- We assume that fire can occur only once on a given plot.
- \( \theta \) = (continuous) risk of total exogenous forest loss on a given plot; or alternatively a small background risk of “dieback” (catastrophic forest loss).
- A property of these processes is that expected time to the first forest fire event, and to forest dieback, on a given plot are \( 1/\lambda \) and \( 1/\theta \).
- \( v \) = per-unit (current or flow) value of services and goods provided by remaining forest. \( v \) can be interpreted either as a local or (biome-wise) average per-period return.
- \( w \) = returns to a given unit of rainforest, via services and products outside of or apart from the forest itself.
\( g = \) (exogenous) rate of growth of forest returns over time, applying to all current terms entering into the forest value. We assume (as a stability condition) that \( r - g > 0 \).

The most direct returns in (1) are the two main value streams per time unit (flow values), \( v(D) \) (current per-time benefits from the forest itself, per unit of forest), and \( w(D) \) (current per-time benefits for the region outside of the forest itself, but created per unit of forest). Both are assumed to be functions of general forest dryness. In addition two main terms express (probabilistic) rates of value changes. The first (which is negative) represents loss of value due to fire risk; this is thus the “fire-induced” loss; the second, \((g-\theta)V\), represents net value growth over time.

(1) can be viewed as valuing the forest in a steady-state with no forest extraction for timber purposes, or with timber extracted sustainably. When deliberate deforestation occurs, and the timber on the respective plot extracted, we will add a net value term \( T \) per unit of \( L \) lost, to represent timber values; see equation (6) below.

\( V_F \) given by (2), represents the forest value for this plot after a fire has occurred, given by a similar asset value equation (where \( F = \) forest lost due to fire on a unit of land):

\[
(2) \quad rV_F = (1 - L - F)[v(D) + w(D)] + (g - \theta)V_F.
\]

(1) and (2) can be solved recursively (as (2) solves for \( V_F \) only):

\[
(3) \quad V = \frac{1}{r - g + \theta} \left[ 1 - \frac{\lambda}{r - g + \lambda + \theta} F \right] [v(D) + w(D)]
\]

\[
(4) \quad V_F = \frac{1}{r - g + \theta} [(1 - L - F)[v(D) + w(D)].
\]

From (3), increased expected damage due to future forest fires reduces the expected present value of the standing forest on a given plot. Forest fires are assumed to damage only the forest, not e.g. crops.
Marginal value of rainforests

I will now consider the economic loss due to a small (exogenous) forest loss. This involves changes in the risk of forest fires when a (small) amount of forest is lost exogenously. It also involves changes in forest dryness, at the macro scale. It is analyzed by taking the (negative of the) derivative of \( V \) with respect to the degree of forest loss, \( L \) in (3), where the forest fire probability, \( \lambda \), is a function of \( L \).

The discounted social value of avoiding one unit of deforestation, on plots which have not yet been subject to forest fire, takes two alternative forms, distinguishing between two types of forest loss, “spontaneous” (“non-intended”) loss, (5), and “deliberate” (“intended”) loss, (6):

\[
\frac{-dV}{dL} = \frac{1}{r-g+\theta} \left[ \frac{r-g+\theta}{(r-g+\lambda+\theta)^2} \lambda_v F + \frac{r-g+\theta}{(r-g+\lambda+\theta)^2} \Lambda_D D' F \right] (v+w) \\
-[V'(D) + W'(D)]D' \frac{V}{v+w}
\]

\[
\frac{-dV}{dL} = \frac{1}{r-g+\theta} \left[ \frac{r-g+\theta}{(r-g+\lambda+\theta)^2} \lambda_v F + \frac{r-g+\theta}{(r-g+\lambda+\theta)^2} \Lambda_D D' F \right] (v+w) \\
-[V'(D) + W'(D)]D' \frac{V}{v+w} - T
\]

In (5), forest is considered to be lost spontaneously due to exogenous factors, e.g. through dieback or illegal logging, assuming that no resource values are captured when forest is lost.

(6) represents the marginal net cost of losing rainforest given clear-cutting through logging, where a net timber value \( T \) per unit of forest area would be realized upon deforestation. We assume, for analytical simplicity, that lost forest is lost forever.

In (5)-(6), the terms containing \( F \) represent impacts of forest loss on overall forest value due to the increased fire risk that occurs when more forest is lost, in turn due to increased forest fragmentation and dryness.

The last terms in both expressions include \( D' \), and represent impacts of forest losses on values (both within and outside of the forest) due to increased general forest dryness, within and outside of the biome. The derivative \( \Lambda_D \) here represents biome-wide impacts (while \( \lambda_v \) represents a plot-specific impact).
Consider factors that influence on marginal forest values in (5) (or (6)). The “effective discount rate” \( r-g+\theta \) is central for this value. A higher exogenous “baseline forest loss rate”, \( \theta \), reduces present discounted forest values, both totally and at the margin.

The first main term of (5) represents the individual plot value which is lost when that piece of forest is lost. It however also represent a biome-wide average value when representing the impact of increased forest dryness, in the second main term (which is a biome-wide phenomenon). Our linearity assumption (whereby overall valuation impacts of forest losses are proportional to the amount of forest lost) leads to substantial simplification as it does not matter whether a marginal forest change occurs on a given plot, or is spread across many or all plots; the marginal valuation results still apply. For impacts of increased dryness, I assume for simplicity that the marginal value \( v'(D) \) is independent of any plot-specific \( v \) at the site where the forest loss occurs.

Note that the plot-specific level of \( v \) could vary substantially across plots. In cases of plot-specific forest losses this is manifested through \( v \) as the first major term on the right-hand sides of (5) and (6), which can be highly variable.

(5) also captures the impacts of endogenous dryness as function of the amount of remaining forest. For any one particular (small) plot of forest, overall dryness might (possibly) be considered exogenous, affected (only or mainly) by macro variables.

The first of the dryness effects, the last term inside the square bracket of (5), represents the macro effects of increased dryness on forest fire incidence when additional forest is lost throughout the macro biome.

The second term related to dryness, after the square bracket, represents direct value loss as the standing value of all forest is reduced by increased dryness, via the expression \( v'(D) \). \( V/(v+w) \) can be interpreted as a (value-related) measure of remaining forest size.

For this model to provide geographically differentiated forest values across the biome, (5)-(6) should express marginal values of forest on small plots of forest. With this interpretation, \( T \) must embed also the value of the deforested land in alternative uses (such as agriculture).

\( V'(D) \) and \( W'(D) \) in (5) represent biome-wide (and not plot-specific) impacts of greater dryness induced by the specific marginal forest loss \( dL \) As long as \( T \) is sufficiently small that – \( dV/dL \) remains positive in (6), eliminating any of the rainforest for the purpose of timber harvesting is not optimal.
(5) gives the total marginal value of saving one additional unit of rainforest. It is natural to associate the first term in the square bracket in (5) (apart from the number 1) with non-fire effects, and the second term with fire effects.

(5) also captures the externality cost of forest loss outside of the forest itself, represented by $W'(D) > 0$, which leads to higher marginal forest value. Less forest increases average forest dryness. This is a standard externality costs associated with deforestation of tropical rainforest. It is specifically relevant for capturing rainfall impacts on economic activities, such as agriculture and hydroelectric production, outside of the forest area.

The marginal forest value (equal to the marginal gain from retaining an additional unit of forest; or avoid a unit of deforestation) is greater than the average per unit forest value, from (3). The reason is the externality costs through greater fire risk, and greater dryness for the remaining forest, when a unit of forest is lost.