University of Oslo / Department of Economics / Nils Framstad

ECON3120/4120 Mathematics 2: Problems for week 42 (2010)

The last parts – i.e. the starred points – are «experimental». Give them your best shot :-)

- Calculate the determinants of Problem 5 (a), 37 (a) and 69 (a).
- Problem 42 (a), (c)
- Problem 128 (a), (c)
- Do problem 48 (b) in three different ways: (I) by identifying $\mathbf{A}$ with a $\mathbf{B}$ as in part (a). (II) by formula. (III) by Gaussian elimination.
- Problem 25
- Problem 58
- Problem 95 (c)
- It is possible to define matrices of infinite order. Consider the infinite matrix $\mathbf{A}$ defined by having ones just at the right of the main diagonal, and zeroes elsewhere, i.e.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & \dots \\ 0 & 0 & 0 & 1 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

  - Compute the matrix products $\mathbf{B} = \mathbf{A}\mathbf{A}'$ and $\mathbf{C} = \mathbf{A}'\mathbf{A}$ (in these cases, matrix multiplication works just like you are used to).
  - Compare $\mathbf{B}$ and $\mathbf{C}$; is this result possible for $n \times n$ matrices?

- Let $\eta(x)$ be defined by the infinite series

$$\eta(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots + \frac{x^n}{n!} + \dots.$$

  - Show that $\eta'(x) = \eta(x)$, and use this to deduce a different expression for η.
  - We want to extend the domain of definition of η to matrices by defining

$$\eta(\mathbf{A}) = \mathbf{I} + \mathbf{A} + \frac{\mathbf{A}^2}{2} + \frac{\mathbf{A}^3}{6} + \dots + \frac{\mathbf{A}^n}{n!} + \dots$$

    * For what kind of matrices $\mathbf{A}$ can $\eta(\mathbf{A})$ be well-defined? What will the order of $\mathbf{B} = \eta(\mathbf{A})$ then be?
    * Define $\mathbf{F}(t) = \eta(\mathbf{A}t)$, for real t. Show that $\frac{\mathrm{d}\mathbf{F}}{\mathrm{d}t} = \mathbf{A}\mathbf{F}(t) = \mathbf{F}(t)\mathbf{A}$.
      (Hint: just like $\frac{\mathrm{d}}{\mathrm{d}t}(mg(t)) = m\frac{\mathrm{d}g}{\mathrm{d}t}$ whenever m does not depend on t, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{M}g(t)) = \mathbf{M}\frac{\mathrm{d}g}{\mathrm{d}t},$$

      whenever $\mathbf{M}$ does not depend on t.)