University of Oslo
Department of Economics

English version

ECON3120/4120 Mathematics 2

Monday, 30 May 2005, 14.30–17.30

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

Give reasons for all your answers.

Grades given run from A (best) to E for passes, and F for fail.

Problem 1

Consider the function f defined by

$$f(x) = x(\ln x)^2, \qquad x > 0$$

(a) Calculate $f'(x)$ and $f''(x)$.

(b) Determine where f is increasing and where f is decreasing. Does f have any global extreme points?

(c) Show that $f(x) \to 0$ as $x \to 0^+$ and that $f'(x) \to \infty$ as $x \to 0^+$.

Problem 2

(a) Find $\displaystyle\lim_{x\to 0} \frac{e^{xt} - 1 - xt}{x^2}$. $\qquad$ (t is a constant.)

(b) Find $\displaystyle\int \frac{e^{4x}}{e^{2x}+1}\, dx$.

(c) Find $\displaystyle\int (\ln x)^2\, dx$.

Problem 3

(a) Calculate the determinant of $\mathbf{A}_t = \begin{pmatrix} 0 & t & 1 \\ 4 & -2 & 8 \\ 1 & 1 & 1 \end{pmatrix}$

(b) Find x, y and z such that

$$\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x & y \\ z & 0 \end{pmatrix} - \begin{pmatrix} x & y \\ z & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ 0 & 1 \end{pmatrix}$$

(Cont.)

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Problem 4

Consider the problem

$$\text{minimize} \quad x^2 + y^2 + z \quad \text{subject to} \quad \begin{cases} x^2 + 2xy + y^2 + z^2 = a \\ x + y + z = 1 \end{cases} \qquad (*)$$

where a is a constant.

(a) Use Lagrange's method to set up necessary conditions for a minimum.

(b) Find the solution of $(*)$ when $a = 5/2$. (You can take it as given that the minimum exists.)

(c) The minimum value in problem $(*)$ depends on a, call it $V(a)$. What is $V'(5/2)$?