University of Oslo
Department of Economics

English

ECON3120/4120 Mathematics 2

Tuesday, 6 June 2006, 14:30–17:30

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

Give reasons for all your answers.

Grades given run from A (best) to E for passes, and F for fail.

Problem 1

Let f be a function of two variables, given by

$$f(x,y) = (x^2 - axy)e^y,$$

where $a \neq 0$ is a constant.

(a) Find the stationary points of f and decide for each of them if it is a local maximum point, a local minimum point or a saddle point.

(b) Let (x^*, y^*) be the stationary point where $x^* \neq 0$, and let $f^*(a) = f(x^*, y^*)$. Find $df^*(a)/da$. Show that if we let $\hat{f}(x,y,a) = (x^2 - axy)e^y$, then

$$\hat{f}_3'(x^*, y^*, a) = \frac{df^*(a)}{da}\,.$$

Problem 2

Consider the equation system

$$\begin{aligned} x + 2y + 3z &= 4 \\ 2x + y + 3z &= 2 \\ x + ty - z &= 4 \end{aligned}$$

where t is a parameter.

(a) Find the solution of the system for all values of t.

(b) Let (x_t, y_t, z_t) be the solution you found in part (a). For what values of t is $2x_t \geq y_t$?

(Cont.)

Problem 3

(a) Find the general solution of the differential equation

$$t(t-1)\dot{x} + x = t^2 e^t, \qquad 0 < t < 1. \tag{D}$$

(*Hint:* The formula $\dfrac{1}{t(t-1)} = \dfrac{1}{t-1} - \dfrac{1}{t}$ may be useful.)

(b) Show that all solutions $x = x(t)$ of (D) tend to 0 as $t \to 0^+$. Show also that only one solution tends to a limit as $t \to 1^-$. Determine this solution and find its limit as $t \to 1^-$.

Problem 4

In a study of the demand for semiconductors one encounters the integral

$$S = \int_0^T e^{-rx}(e^{g(T-x)} - 1)\,dx,$$

where T, r and g are positive constants.

(a) Show that

$$r(e^{gT} - e^{-rT}) - (r+g)(1 - e^{-rT}) = r(r+g)S. \tag{$*$}$$

(b) The equation $(*)$ defines T as a function of g, r and S. Find an expression for $\partial T/\partial g$.