University of Oslo
Department of Economics

ECON3120/4120 Mathematics 2

Wednesday 26 May 2004, 9.00–12.00

There are 2 pages of problems.

All written or printed material may be used, as well as pocket calculators.

State reasons for all your answers.

Grades given run from A (best) to E for passes, and F for fail.

Problem 1

Consider the function f defined by

$$f(x) = 4 - x + \ln(x-1), \qquad x > 1.$$

(a) Compute $f'(x)$ and $f''(x)$.

(b) Determine where f is increasing and where f is decreasing. Does f have any global extreme points?

(c) Find $\lim_{x\to 1^+} f(x)$ and $\lim_{x\to\infty} f(x)$.

(d) Show that the equation $f(x) = 0$ has exactly two roots. (You are not supposed to find these roots.)

Problem 2

Calculate the integrals

$$\text{(i)} \int (x^3 + 2x)^2\, dx \qquad\qquad \text{(ii)} \int_0^{\sqrt{8}} \frac{x}{(1+x^2)^a}\, dx \quad (a \neq 1)$$

Problem 3

Let $\mathbf{A}$ be the matrix $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) Calculate the matrix product $\mathbf{A'A}$ and the determinant $|\mathbf{A'A}|$.

(b) Does $\mathbf{A'A}$ have an inverse? Find $(\mathbf{A'A})^{-1}$ if it exists.

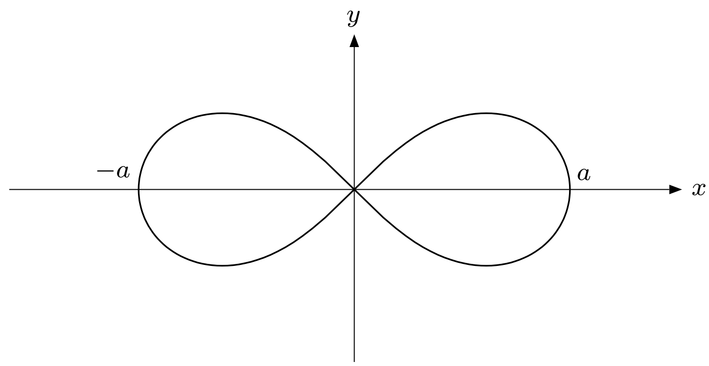
(Cont.)

Problem 4

In one of his papers, the Norwegian mathematician Niels Henrik Abel studied the curve shown in the figure below, known as a *lemniscate*. It is given by the equation

$$(x^2 + y^2)^2 = a^2(x^2 - y^2)$$

where a is a positive constant.



(a) Find an expression for the slope of the tangent to this curve at a point (x, y) where $y \neq 0$.

(b) Determine those points on the curve where the tangent is horizontal (i.e. parallel to the x-axis).

Problem 5

Write down the necessary Lagrange conditions for a point (x, y) to solve the problem

$$\text{maximize } f(x, y) = x + xy \quad \text{subject to} \quad y + x^2 e^y = 1.$$

Show that $(x_0, y_0) = (1, 0)$ satisfies these conditions, and find the corresponding value of the Lagrange multiplier λ.