University of Oslo
Department of Economics

**English version**

# ECON3120/4120 Mathematics 2

Wednesday, 24 November 2004, 14.30–17.30

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

Give reasons for all your answers.

Grades given run from A (best) to E for passes, and F for fail.

## Problem 1

Let $f(x) = (x^2 - a)e^{-bx}$, where $a$ and $b$ are constants, $b \neq 0$.

(a) Compute $f'(x)$ and $f''(x)$.

(b) Put $a = 5$ and $b = 1/2$. Find the local and global extreme points of $f$, if any.

(c) Calculate $\displaystyle\int_0^\infty (x^2 - 5)e^{-x/2}\,dx$.

## Problem 2

(a) Evaluate the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & a \\ 1 & 2 & b \end{vmatrix}$.

(b) For what values of the parameters $a$, $b$, and $c$ will the equation system

$$\begin{aligned} x + y + z &= c \\ x + 2y + az &= 2c \\ x + 2y + bz &= 2 \end{aligned}$$

have (i) a unique solution, (ii) several solutions, (iii) no solutions?

(Cont.)

**Problem 3**

Consider the problem

$$(*) \quad \text{maximize } f(x,y,z) = x + 2y + \ln(1+z) \quad \text{subject to} \quad x^2 + y^2 - az = 0,$$

where $a$ is a constant.

(a) Write down the necessary Lagrange conditions for a point $(x, y, z)$ to solve problem $(*)$.

(b) Solve problem $(*)$ when $a = -3$. (Assume that there exists a solution.)

(c) Show that $(*)$ does not have any solutions when (i) $a = 0$, (ii) $a = 1$.

**Problem 4**

(a) Show that, if $\alpha > 0$, there is no $3 \times 3$ matrix $\mathbf{C}$ such that $\mathbf{C}^2 = -\alpha \mathbf{I}_3$.

(b) Use the result in (a) to show that there is no $3 \times 3$ matrix $\mathbf{B}$ such that $\mathbf{B}^2 + \mathbf{B} + \mathbf{I}_3 = \mathbf{0}$.
(*Hint:* What is $(\mathbf{B} + \frac{1}{2}\mathbf{I}_3)^2$ ?)