University of Oslo / Department of Economics **English**

ECON3120/ECON4120 Mathematics 2

Tuesday June 3 2008, 09:00–12:00

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

Give reasons for all your answers.

Grades given run from A (best) to E for passes, and F for fail.

Problem 1 Let for each d the matrix $\mathbf{A}_d$ be given by

$$\mathbf{A}_d = \begin{pmatrix} d^2 & 1 & 2 & 3 & 4 \\ 0 & -1 & 5 & 6 & 7 \\ 0 & 0 & d^2+1 & 8 & 9 \\ 0 & 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 0 & d^2 \end{pmatrix}$$

(a) Calculate $|\mathbf{A}_d|$ and show that the nth power $\mathbf{A}_d^n$ has an inverse if and only if $d \neq 0$.

(b) Consider the equation system

$$\mathbf{A}_d^{2008}\,\mathbf{x} = \begin{pmatrix} d \\ d \\ d \\ d \\ d \end{pmatrix}$$

where $\mathbf{x}$ is a vector $(x_1, \ldots, x_5)'$ of five unknowns and $\mathbf{A}_d^{2008}$ is the 2008th power of $\mathbf{A}_d$.

(i) For what values of d will the equation system have a *unique* solution?

(ii) Show that there always is a solution.

You may use the result given in part (a) no matter whether you managed to show it or not.

Problem 2 The equation system

$$\begin{aligned} x\,\ln u + v\,\ln y &= 0 \\ -e^y + uv + e^{ux} &= 0 \end{aligned}$$

defines (u, v) as continuously differentiable functions of (x, y) near the point $(x, y, u, v) = (a, a, 1, 0)$, for a suitable positive constant a.

Differentiate the system (i.e. calculate differentials), and find expressions for $v'_x(a, a)$ and $v'_y(a, a)$.

Problem 3 Let $b > 0$ be a constant. Consider the differential equation

$$\ddot{y} = \dot{y} + e^{-bt} \tag{*}$$

Thus, if $z = \dot{y}$, then z satisfies the differential equation

$$\dot{z} = z + e^{-bt} \tag{**}$$

(a) Find the general solution of (**).

(b) Use the general solution z of (**) to find the particular solution y of (*) with the property that $\lim_{t \to \infty} y(t) = 1$.

Problem 4 Let

$$f(x, y) = e^{xy^2} - Ax^2y - 1,$$

where $A \neq 0$ is a constant. Consider the problem

$$\max_{(x,y)} f(x, y) \qquad \text{subject to} \qquad \begin{cases} x & \geq 0 \\ y & \geq 0 \\ x + y & \leq A^2 \end{cases} \tag{P}$$

(a) State the Kuhn-Tucker conditions associated with the problem (P), and show that they are satisfied at the origin.

(b) You can take for granted that problem (P) does not have a solution on the axes. Use this to show that at the solution point, one and only one of the Lagrange multipliers is $\neq 0$.
Hint: Show first that f has no stationary point outside the axes.