University of Oslo / Department of Economics **English**

# ECON3120/ECON4120 Mathematics 2

Thursday December 15 2011, 14:30–17:00

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

Grades given run from A (best) to E for passes, and F for fail.

You are required to state reasons for all your answers.

**Problem 1** Define for each number $t$ the matrix $\mathbf{A}_t$ by

$$\mathbf{A}_t = \begin{pmatrix} t & 1 & 0 \\ 1 & t & 1 \\ 0 & 1 & t \end{pmatrix}.$$

(a) Calculate the determinant of $\mathbf{A}_t$.

(b) For which value(s) of $t$ does the equation system

$$\mathbf{A}_t\,\mathbf{x} = \begin{pmatrix} \sqrt{2} \\ 1 \\ \sqrt{2} \end{pmatrix}$$

have (i) unique solution, (ii) no solution, (iii) several solutions?

**Problem 2** Let $a \in (0,1)$, $b \in (0,1)$, $A > 0$, $B > 0$ and $k \geq 0$ be constants, and define for nonnegative $x, y$,

$$F(x,y) = Ax^a + By^b + kxy$$

(a) A level curve $F(x,y) = C$ $(> 0)$ will define $y$ as a function of $x$. Find an expression for the elasticity $\mathrm{El}_x y$ in terms of $x$, $y$ and the constants $a, A, b, B, k$ (i.e., not involving $C$).

(b) In this part, let $k = 0$. Show that the elasticity of substitution between $y$ and $x$ can be written as

$$\frac{aAx^a + bBy^b}{aAx^a + bBy^b - abF(x,y)}$$

**Problem 3** Define $f(x, y) = x^4 - x^2 + y^2$, let $K$ be a positive constant, and consider the problem

$$\min\ f(x, y) \qquad \text{subject to} \quad x^2 + 2y^2 = K \tag{P}$$

(a) Explain why (P) has a solution, and state the Lagrange conditions associated to the problem.

(b) Find all points which satisfy the Lagrange conditions with $x \geq 0,\ y \geq 0$.

(c) The optimum will be one of the points from part (b). The optimal value of $f(x, y)$ is a function $V(K)$. Calculate $V'(1)$ in two ways: (i) by means of the Lagrange multiplier, and (ii) by evaluating $V$ and differentiating.

**Problem 4** Let $k > 0$ be a constant. Consider the differential equation

$$2\, x(t)\, \dot{x}(t) = \big([x(t)]^2 + k\big)\, t\, e^{2t}$$

(a) Find the general solution.

(b) Find the particular solution for which $x(1) = 2$.