University of Oslo / Department of Economics **English**

ECON3120/4120 Mathematics 2

Tuesday May 29 2012, 09:00–12:00

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

You are required to state reasons for all your answers. Throughout the problem set, you are permitted to use without proof information from a previous part, regardless of whether you managed to solve it or not.

Grades given run from A (best) to E for passes, and F for fail.

Problem 1 Define for each value of u the matrix

$$\mathbf{A}_u = \begin{pmatrix} 0 & 1 & 2 \\ 3u & u+3 & u^3 \\ 0 & u-3 & u-4 \end{pmatrix}$$

(a) Show that $|\mathbf{A}_u| = 0$ if and only if $u = 0$ or $u = 2$.

(b) Consider the equation system (where $\mathbf{x} \in \mathbf{R}^3$ is the unknown)

$$\mathbf{A}_u \mathbf{x} = \begin{pmatrix} u \\ 2u \\ 3u \end{pmatrix}$$

Decide, for each value of u, whether the system has (i) no solution, (ii) one solution, or (iii) more than one solution.

Problem 2 In this problem, you shall take as given without proof that the equation system

$$x + v + e^{xu} - v^5 y = 1$$
$$\ln(1 + xuv) + x + ve^{uy} = e$$

defines u and v as continuously differentiable functions of x and y around the point P with coordinates $(x, y, u, v) = (0, 1, 1, 1)$.

(a) Differentiate the equation system (i.e., calculate differentials).

(b) Calculate $u'_x(0, 1)$ and $u'_y(0, 1)$.

Problem 3

(a) Evaluate the integrals

$$\text{(i)} \int x^3 e^x \, dx, \qquad \text{(ii)} \int_0^1 e^{y^{1/4}} \, dy, \qquad \text{(iii)} \int_2^e \frac{(\ln(\ln z))^3}{z} \, dz$$

(Hint: In (ii) and (iii), a substitution will lead to an integral where you can use (i).)

(b) Throughout this part, assume that t is nonnegative. Show that

$$\int \frac{1}{(t+1)(t+2)^2} \, dt = \frac{1}{t+2} + \ln \frac{t+1}{t+2} + C$$

and use this to find the particular solution of the differential equation

$$\dot{x}(t) = \frac{x(t)}{(t+1)(t+2)^2} \qquad \text{with} \qquad x(0) = 1/\sqrt{e}.$$

Problem 4 Let $k > 0$ be a constant, and consider the function

$$g(x) = k \cdot (1 - e^{-kx}) + x \ln(1 + x^2) - x^2$$

(a) Calculate

$$\lim_{x \to +\infty} \frac{\ln(1 + x^2)}{x}$$

and use this to show that

$$\lim_{x \to +\infty} g(x) = -\infty$$

(Hint: $x \ln(1 + x^2) - x^2 = x^2 \cdot \left[\frac{\ln(1+x^2)}{x} - 1\right]$.)

(b) Use part (a) and the fact that $g'(0) > 0 = g(0)$ to show that g has (i) some zero $x_0 > 0$, and (ii) some stationary point $\bar{x} \in (0, x_0)$.
(You are not supposed to compute neither x_0 nor $\bar{x}$.)

(c) The stationary point $\bar{x}$ of part (b) will be a function $h(k)$. Put $G(k) = g(h(k))$. Find an expression for $G'(k)$.