University of Oslo / Department of Economics **English**

ECON3120/4120 Mathematics 2

June 4 2013, 09:00–12:00

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

Grades given run from A (best) to E for passes, and F for fail.

- You are required to state reasons for all your answers.
- You are permitted to use any information stated in an earlier letter-enumerated item (e.g. "(a)") to solve a later one (e.g. "(c)"), regardless of whether you managed to answer the former. A later item does not necessarily require answers from or information given in a previous one.

Problem 1 Define for each real number t the matrix $\mathbf{A}_t$ and the vector $\mathbf{b}_t$ by

$$\mathbf{A}_t = \begin{pmatrix} t & 3 & 0 \\ 1 & t+1 & 3 \\ 0 & 1 & t \end{pmatrix} \qquad \text{and} \qquad \mathbf{b}_t = \begin{pmatrix} t \\ t^2 \\ t^3 \end{pmatrix}$$

(a) (i) Find real numbers p, q such that $\mathbf{A}_s + \mathbf{A}_t = p\mathbf{A}_q$.

(ii) Calculate the determinant of $\mathbf{A}_t$.

(b) Find those t for which there is a solution (one or more!) of the equation system

$$\mathbf{A}_t\,\mathbf{x} = \mathbf{b}_t$$

(where $\mathbf{x} = (x_1, x_2, x_3)'$ is the unknown vector.)

(c) Is there any t such that the equation system

$$\mathbf{A}_t^{2013}\,\mathbf{x} = \mathbf{b}_t$$

has *infinitely* many solutions? ($\mathbf{A}_t^{2013}$ denotes the 2013th power.)

Problem 2 Consider the function $F(t) = t^t - \dfrac{1}{\sqrt[3]{3}}$ defined for $t > 0$.

(a) Show that

$$\int t^t(1 + \ln t)\,dt = F(t) + C$$

and use this to find

$$\int_0^{\ln 2} (z+1)e^{z(1+e^z)}\,dz.$$

(*Hint:* for the latter, use the substitution $t = e^z$.)

(b) How many zeroes does F have?
(You are not asked to calculate any, but observe that $F(1/3) = 0$ and that $F'(1/3) < 0$.)

(c) Use part (a) to find the particular solution which passes through $(t_0, x_0) = (2, 2)$ of the differential equation

$$3\dot{x}(t) = \frac{t^t(1 + \ln t)}{\big(x(t)\big)^2}$$

Problem 3 Let N be a positive integer (i.e. $1, 2, ...$) and let f be the function

$$f(x, y) = xy^{N+1} - (x+1)\ln(x+1)$$

defined for $x > -1$, all y.

(a) Find and classify the stationary points of f.

(b) Does f have any global maximum or minimum?

In the following, consider the problem

$$\max\ f(x,y) \qquad \text{subject to} \quad \begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq 1 - x \end{cases} \tag{P}$$

(c) Decide whether this problem has a solution or not, and state the associated Kuhn–Tucker conditions.

(d) Put $x = 0$ in the Kuhn–Tucker conditions. For what $y \in [0, 1]$ will the Kuhn–Tucker conditions be satisfied for the point $(0, y)$?

(e) Show that the Kuhn–Tucker conditions cannot be satisfied when $0 < x < 1 - y$.