ECON3120/4120 Mathematics 2

December 10th 2013, 1430–1730.

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

Grades given run from A (best) to E for passes, and F for fail.

- You are required to state reasons for all your answers.
- You are permitted to use any information stated in an earlier letter-enumerated item (e.g. "(a)") to solve a later one (e.g. "(c)"), regardless of whether you managed to answer the former. A later item does not necessarily require answers from or information given in a previous one.

Problem 1 In this problem, let t be a constant and consider the linear equation system $\mathbf{A}_t\mathbf{x} = \mathbf{b}$ in the unknown vector $\mathbf{x}$, where

$$\mathbf{A}_t = \begin{pmatrix} 12-3t & 2 & 2 \\ 1 & 0 & 2 \\ 12 & 2-t & 2-t \end{pmatrix}, \qquad \text{and} \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 0 \\ 12 \end{pmatrix}.$$

(a) Find q such that $\mathbf{A}_t$ has determinant equal to $q \cdot t(t-6)$.

(b) For what values of the constant t will the equation system $\mathbf{A}_t\mathbf{x} = \mathbf{b}$ have (i) unique solution, (ii) no solution, (iii) several solutions?

Problem 2 Let $p > 0$ be a constant and let $g(x) = (2x+p)e^{-px} + pxe^{-2px}$.

(a) (i) Find $\lim_{x\to-\infty} g(x)$ or show that it does not exist.

(ii) Show that $\lim_{x\to+\infty} g(x) = 0$.

(b) Show that there exists an $\hat{x} < 0$ such that $g(\hat{x}) = 0$.

(c) Find $\int g(x)\,dx$ and decide whether $\int_p^\infty g(x)\,dx$ converges.

Problem 3 Let r be a positive constant. Consider the problem

$$\text{minimize} \quad e^{2y+(1-2r)x-r} - \ln(x+y+r/2) \qquad \text{subject to} \quad x+2y=r$$

Write out the associated Lagrange conditions, and show that they are equivalent to

$$2r(x+2r)e^{-2rx}+1=0.$$

(Hint: Eliminate the multiplier and then y.)

Problem 4

(a) (i) Show that

$$\int \Big[2t(\ln t - 1) - \ln t\Big] e^{-2t}\,\mathrm{d}t = t(1-\ln t)e^{-2t} + C$$

(ii) Show that $t \cdot (1-\ln t)$ is a particular solution of the differential equation

$$\dot{x}(t) = 2x(t) + 2t(\ln t - 1) - \ln t, \qquad t>0 \tag{$*$}$$

(b) Find the general solution of the differential equation $(*)$. (Hint: Use part (a).)

(c) One particular solution of $(*)$ passes through the point where $(t,x)=(1,3)$. Find the equation for the tangent line at that point.

Problem 5 An exam problem in ECON5155 in December 2012, led to an equation system of the form

$$\begin{aligned}(e^{tu}+e^{-tu})v - u + s &= 0\\ (e^{tu}-e^{-tu})tv &= 1\end{aligned}$$

which defines u and v as continuously differentiable functions of s and t around the point P where $s=-2(e+3)/(e-1)$, $t=1/4$, $u=2$ and $v=4\sqrt{e}/(e-1)$. (You are not supposed to show this.)

(a) Differentiate the equation system (i.e. calculate differentials).

(b) Use the differentiated system to find the value of $\partial u/\partial s$ at P.
(You *are required to* use the differentiated system, even though there might be other methods.)