University of Oslo / Department of Economics **English version**

ECON3120/4120 Mathematics 2

May 23rd 2014, 1430–1730.

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

Grades given run from A (best) to E for passes, and F for fail.

- You are required to state reasons for all your answers.
- You are permitted to use any information stated in an earlier letter-enumerated item (e.g. "(a)") to solve a later one (e.g. "(c)"), regardless of whether you managed to answer the former. A later item does not necessarily require answers from or information given in a previous one.

Problem 1 Define the matrix $\mathbf{A}$ and for each real number t the matrix $\mathbf{M}_t$ by

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix}, \qquad \mathbf{M}_t = \begin{pmatrix} 0 & 1 & -1 \\ 2t & -t & 0 \\ -t & -t & 1 \end{pmatrix}$$

(a) i) Calculate $\mathbf{S}_t = t\mathbf{A} + \mathbf{M}_t$.

ii) Calculate $\mathbf{P}_t = \mathbf{A}\mathbf{M}_t$.

iii) For what t does $\mathbf{Q}_t = \mathbf{M}_t\mathbf{A}$ have an inverse?

(b) Find a t such that the vector

$$\mathbf{x} = \mathbf{M}_t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

is a solution of the equation system

$$\mathbf{A}\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

(c) Let t be the number from part (b).

i) Why is it so that the vector $\mathbf{x} = \mathbf{M}_t\ (a,\ b,\ c)'$ is a solution of the equation system $\mathbf{A}\mathbf{x} = (a,\ b,\ c)'$ no matter what a, b and c?

ii) Could there be other solutions, for any values of a, b, c?

Problem 2

(a) Calculate the integral $\displaystyle\int_1^T te^{4t}\,dt$.

(b) Find the general solution of the differential equation $\dot{x} = 3x^2\,te^{4t}$.

(c) Find the particular solution of the differential equation $\dot{x} = 3x + te^{7t}$, such that $x(1) = 2$.

Problem 3 Let $f(x,y) = \ln x + xy - y^2 - 2y\sqrt{3}$.

(a) Find and classify the two stationary points of f.

(b) Does f have any *global* extreme point(s)?

Consider the nonlinear programming problem

$$\max f(x,y) \qquad \text{subject to} \qquad x + y \leq 1, \qquad y \geq 0 \tag{P}$$

(c) State the Kuhn–Tucker conditions and verify that they are satisfied at the point $(x,y) = (1,0)$.

(d) Show that $(x,y) = (1,0)$ solves problem (P).

(e) Approximately how much will the optimal value be reduced if the constraint $y \geq 0$ is tightened to require $y \geq 1/200$?

Problem 4 Assume that $h(x,y)$ is homogeneous of degree k, and let $H(x,y) = h(x,y) + k$. Find all k such that H is homogeneous, and find all k such that H is homothetic.