University of Oslo
Department of Economics

Compulsory term paper 1 in ECON3120/4120 Mathematics 2

Handed out: Thursday 17 March 2005

To be handed in on: Wednesday 6 April 2005

Hand in at: To be announced. Keep an eye on the ECON4120 home page.

Further instructions:

- This term paper is **compulsory**.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- You must use a preprinted front page, which you will find at *http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc*
- It is important that the term paper is delivered by the deadline (see above). Term papers delivered after the deadline **will not be read or marked.**[*)]
- All term papers must be delivered at the place to be announced on the ECON4120 home page. You must not deliver your term paper to the course teacher or send it by e-mail. If you want to hand in your term paper **before** the deadline, please contact the department office on the 12th floor.
- If your term paper is not accepted as satisfactory, you will be allowed a new attempt with a very short deadline. If you still do not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that it will not count as an attempt.

*) If you believe that you have good a reason for not meeting the deadline (e.g. illness), you should discuss the matter with your course teacher and seek a formal extension. Normally, an extension will be granted only when there is a good reason backed by supporting evidence (e.g. a medical certificate).

Oppgave 1

Consider the function

$$g(x) = \ln[x \ln(x^{2n})]$$

where n is a natural number.

(a) Where is g defined? (Note that $\ln(x^{2n}) = 2n \ln|x|$, in case you want to use that.) Show that $g(-e^{-1}) = \ln 2 + \ln n - 1$.

(b) Find $g'(x)$, and determine where g is increasing/decreasing.

(Cont.)

Oppgave 2

Define the function f by $f(x) = 4\,\dfrac{(\ln x)^2}{x}$ for all $x > 0$.

(a) Calculate $f'(x)$ and $f''(x)$.

(b) Find the local extreme points and inflection points of f, if any.

(c) Sketch the graph of f.

(d) Calculate the area below the graph of f over the interval $\left[1, e^3\right]$.

(e) Discuss the number of solutions of the equation $f(x) = c$ for different values of c.

Oppgave 3

Find the following integrals:

(a) $\displaystyle\int_1^5 \frac{x+3}{\sqrt{3x+1}}\,dx$

(b) $\displaystyle\int x^2(e^x - 1)\,dx$

(c) $\displaystyle\int \frac{\sqrt{x}}{\sqrt{x}-1}\,dx$

Oppgave 4

For every real number t, we define the matrix $\mathbf{A}_t$ by

$$\mathbf{A}_t = \begin{pmatrix} 1 & t & 0 & t \\ 1 & 0 & t & 0 \\ 0 & 1 & -t & 1 \\ 1 & t-1 & 1 & 0 \end{pmatrix}$$

(a) Calculate $|\mathbf{A}_t|$. For what values of t will the homogeneous equation system $\mathbf{A}_t\mathbf{x} = \mathbf{0}$ have nontrivial solutions?

(b) Let $\mathbf{B}$ be a given $n \times n$ matrix. An $n \times n$ matrix $\mathbf{P}$ is said to *commute* with $\mathbf{B}$ if $\mathbf{BP} = \mathbf{PB}$. Show that if $\mathbf{P}$ og $\mathbf{Q}$ commute with $\mathbf{B}$, then $\mathbf{PQ}$ will also commute with $\mathbf{B}$.

(c) Suppose that $\mathbf{Q}$ commutes with $\mathbf{B}$ and that $\mathbf{Q}^{-1}$ exists. Will $\mathbf{Q}^{-1}$ then necessarily commute with $\mathbf{B}$?