University of Oslo
Department of Economics

Compulsory term paper 2 in ECON3120/4120 Mathematics 2

Handed out: Monday 18 April 2005

To be handed in on: Thursday 28 April 2005

Hand in at the department office, 12th floor.

Further instructions:

- This term paper is **compulsory**.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- You must use a preprinted front page, which you will find at *http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc*
- It is important that the term paper is delivered by the deadline (see above). Term papers delivered after the deadline **will not be read or marked.***)
- All term papers must be delivered at the place given above. You must not deliver your term paper to the course teacher or send it by e-mail. If you want to hand in your term paper **before** the deadline, please contact the department office on the 12th floor.
- If your term paper is not accepted as satisfactory, you will be allowed a new attempt with a very short deadline. If you still do not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that it will not count as an attempt.

*) If you believe that you have good a reason for not meeting the deadline (e.g. illness), you should discuss the matter with your course teacher and seek a formal extension. Normally, an extension will be granted only when there is a good reason backed by supporting evidence (e.g. a medical certificate).

Oppgave 1

Let $f(x) = (x^2 - a)e^{-bx}$, where a and b are constants, $b \neq 0$.

(a) Compute $f'(x)$ and $f''(x)$.

(b) Put $a = 5$ and $b = 1/2$. Find the local and global extreme points of f, if any.

(c) Calculate $\displaystyle\int_0^\infty (x^2 - 5)e^{-x/2}\,dx$.

(Cont.)

Oppgave 2

(a) Evaluate the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & a \\ 1 & 2 & b \end{vmatrix}$.

(b) For what values of the parameters a, b, and c will the equation system

$$\begin{aligned} x + \ \ y + \ \ z &= \ c \\ x + 2y + az &= 2c \\ x + 2y + bz &= \ 2 \end{aligned}$$

have (i) a unique solution, (ii) several solutions, (iii) no solutions?

Oppgave 3

Consider the problem

(∗) maximize $f(x, y, z) = x + 2y + \ln(1 + z)$ subject to $x^2 + y^2 - az = 0$,

where a is a constant.

(a) Write down the necessary Lagrange conditions for a point (x, y, z) to solve problem (∗).

(b) Solve problem (∗) when $a = -3$. (Assume that there exists a solution.)

(c) Show that (∗) does not have any solutions when (i) $a = 0$, (ii) $a = 1$.

Oppgave 4

(a) Show that, if $\alpha > 0$, there is no 3×3 matrix $\mathbf{C}$ such that $\mathbf{C}^2 = -\alpha\mathbf{I}_3$.

(b) Use the result in (a) to show that there is no 3×3 matrix $\mathbf{B}$ such that $\mathbf{B}^2 + \mathbf{B} + \mathbf{I}_3 = \mathbf{0}$.
(*Hint:* What is $(\mathbf{B} + \frac{1}{2}\mathbf{I}_3)^2$?)