University of Oslo
Department of Economics

Compulsory term paper 2 in ECON3120/4120 Mathematics 2

Handed out: Wednesday 19 April 2006.

To be handed in on Thursday 4 May 2006.

Hand in at the department office, 12th floor (before 2:00 pm).

Further instructions:

- This term paper is **compulsory**.
- This paper will NOT be given a grade that counts towards your final grade for this course. A possible grade is meant only for your guidance.
- You must use a preprinted front page, which you will find at *http://www.oekonomi.uio.no/info/EMNER/Forside_obl_eng.doc*
- It is important that the term paper is delivered by the deadline (see above). Term papers delivered after the deadline **will not be read or marked.**[*)]
- All term papers must be delivered at the place given above. You must not deliver your term paper to the course teacher or send it by e-mail. If you want to hand in your term paper **before** the deadline, please contact the department office on the 12th floor.
- If your term paper is not accepted as satisfactory, you will be allowed a new attempt with a very short deadline. If you still do not succeed, you will not be permitted to take the exam in this course. You will then be withdrawn from the exam, so that it will not count as an attempt.

*) If you believe that you have good a reason for not meeting the deadline (e.g. illness), you should discuss the matter with your course teacher and seek a formal extension. Normally, an extension will be granted only when there is a good reason backed by supporting evidence (e.g. a medical certificate).

Problem 1

Consider the problem

$$\text{maximize } f(x,y,z) = x + \ln(1+z) \quad \text{subject to } \begin{cases} x^2 + y^2 = 1, \\ x + y + z = 1. \end{cases}$$

(a) Solve the problem by Lagrange's method. You may take it as given that there exists a maximum point.

(b) Find an approximate value for the change in the maximum value of $f(x,y,z)$ if the second constraint in the problem is changed to $x + y + z = 1.02$.

(Cont.)

Problem 2

The equation system

$$u^2 + xe^{v^2} - y = 2$$
$$u + e^{v-y^2} + x = 2 \tag{S}$$

defines u and v as differentiable functions of x and y around the point $(x, y, u, v) = (0, -1, 1, 1)$.

(a) Differentiate the equation system.

(b) Find the values of u'_x, u'_y, v'_x, and v'_y at the given point.

(c) Find (approximate) values of u og v that satisfy (S) when $x = 0.02$, $y = -1.01$.

Problem 3

Consider the matrix $\mathbf{A} = \begin{pmatrix} a & a-1 & a \\ a-1 & 1 & 0 \\ a & 0 & a \end{pmatrix}$.

(a) Calculate the determinant of $\mathbf{A}$ for all values of a.

(b) For what values of a and b will the equation system

$$\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b \\ 0 \\ 1 \end{pmatrix}$$

have an infinite number of solutions?

(c) For what values of a is there a matrix $\mathbf{B}$ such that $\mathbf{AB} = \mathbf{A} + \mathbf{B}$?

Problem 4

Let $f(x) = e^{-x} + a\ln(1 + x^2) - 1$, where a is a positive constant.

(a) Calculate $f'(x)$ and show that $f(0) = 0$ and $f'(0) < 0$. What happens to $f(x)$ as $x \to \infty$? Show that there must be at least one value of x for which $f(x) = 0$.

(b) It can be shown (but you are not supposed to do it!) that the equation $f(x) = 0$ has exactly one positive solution $x = x_a$ (which depends on a). Show that x_a decreases as a increases. Determine the limit $\lim_{a\to\infty} x_a$.

> Remember that you are supposed to work independently! It is OK, and often instructive, to discuss the problems with your fellow students, but direct copying is forbidden!

Corrigendum: A word went missing in the last sentence of Problem 4(a) as originally published. The correct version is: ... there must be at least one **positive** value of x for which ...