UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Assignment in: **ECON4130 – Statistics 2**

Handed out: Monday, October 8, 2007

To be delivered by: Monday, October 22, 2007 **not later than 2:00 p.m.**

Place of delivery: Department office, 12^{th} floor

Further instructions:

- This assignment is part of the **portfolio assessment**. Candidates who have passed the portfolio assessment in a previous semester, do not have the right to hand in the assignments again. This is so, even if the candidate did not pass the exam.
- **Note:** The students can feel free to discuss with each other how to solve the problems, but each student is supposed to formulate her/his own answers. Only single-authored papers are accepted, and papers that for all practical purposes are identical will not be approved.
- If one of the assignments is not accepted, you will be given a new attempt. In order to sit in for the exam, all three assignments must be approved. If no, you will be withdrawn from the exam, so that this will not be an attempt.
- If a student believes that she or he has a good cause not to meet the deadline (e.g. illness) she or he should discuss the matter with the course teacher and seek a formal extension. Normally extension will only be granted when there is a good reason backed by supporting evidence (e.g. medical certificate).

ECON 4130 Statistics 2
Autumn 2007
Tapas Kundu

Assignment 2

Exercise 1

a. Show that the moment-generating functions of Poisson (λ) distribution is $e^{\lambda\left(e^{t}-1\right)}$.

b. Show that the moment-generating function of Binomial (n,p) distribution is $\left(1-p+pe^{t}\right)^{n}$.

c. Let $X_1,\cdots,X_n,\cdots$ be a sequence of random variables where $X_k \backsim$ Binomial(k,p), $k=1,2,3,\cdots$. Further, assume that $np=\lambda$. Applying Theorem 2 in 'the lecture notes to Chapter 5', show that $X_n \xrightarrow{D} Y$ where $Y \backsim$ Poisson(λ).

[Hint: Find the moment generating function of the binomial distribution. Take limit. You have to use the following limiting property of real numbers: $\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{n}=e^{a}$]

Exercise 2

a. Suppose that $U \backsim$ Uniform $[0,1]$. Let $\lambda>0$. Show that $Y=-\frac{1}{\lambda}\ln(U)$ is exponentially distributed with parameter λ (i.e., $Y \backsim$ exponential(λ)).

b. Using *a.* there is a simple way to simulate observations from a geometric distribution. Let $X \backsim$ geometric(p), with density function

$$f(x)=p(1-p)^{x-1},\ x=1,2,3,\cdots$$

We want to simulate n independent observations of X for a given p. As a first step, assume that $Y \backsim$ exponential(λ). Show that

$$P(x-1<Y\leq x)=p(1-p)^{x-1}=P(X=x)$$

where λ is chosen as $\lambda=-\ln(1-p)$, and $x=1,2,3,\cdots$.

c. Introduce "[ ]" as a notation for truncation upwards, i.e., $[a]$ means the smallest integer larger or equal to a. For example, $[3.2]=4$ while $[3]=3$. Let $U_i \backsim iid$ and uniform $[0,1]$, for $i=1,2,\ldots,n$. Show that

$$X_i=\left[\frac{\ln(U_i)}{\ln(1-p)}\right]\backsim \text{geometric}(p),\ i=1,2,\ldots,n.$$

Exercise 3

a. Suppose that X is Pareto distributed with density function

$$f(x)=\begin{cases}\dfrac{\theta b^{\theta}}{x^{\theta+1}} \text{ for } x>b\\ 0 \text{ otherwise}\end{cases},\qquad (1)$$

with parameters $b > 0$ and $\theta > 0$. Show that

$$E(X^r) = \begin{cases} b^r \frac{\theta}{\theta - r} \text{ for } 0 < r < \theta \\ \text{does not exist for } r \geq \theta \end{cases}.$$

b. Let $X_1, X_2, \ldots$ be iid and pareto distributed as in (1) with parameters (b, θ), where b is known. Define

$$\hat{\theta} = \frac{n}{\sum_{i=1}^{n} \ln\left(\frac{X_i}{b}\right)}$$

Show that $\hat{\theta}$ is a consistent estimator of θ.

[Hint for *b*. Define $Y_i = \ln\left(\frac{X_i}{b}\right)$. Find $E(Y_i)$. Use the results from the section on 'convergence in probability in lecture notes to chapter 5' to show $\hat{\theta} \xrightarrow{P} \theta$.]