

Exercises for seminar week 42

Rice, chapter 4: No. 75, 83 (use the mgf), 85 (see hint), 100 (read section 4.6 in Rice and (A4-5) in appendix 1 in Lecture notes to Rice chapter 5".)

Rice, chapter 5: No. 4, 12

Hint for ex 4:85: Remember the sum of a geometric series:

$$1 + a + a^2 + a^3 + \dots = \sum_{i=0}^{\infty} a^i = \frac{1}{1-a} \quad \text{for all numbers, } a, \text{ such that } |a| < 1.$$

A common factor in such a series can be taken outside the sum as for finite sums:

$$\sum_{i=0}^{\infty} ca^i = c \sum_{i=0}^{\infty} a^i$$

Hint for ex 5:4: Remember from the basic course, Stat I, that if X is poisson distributed with parameter, m , $X \sim \text{pois}(m)$, we know that $E(X) = \text{var}(X) = m$, and if $m \geq 10$ (about), then X is approximately normally distributed,

$$X \underset{\text{approximately}}{\sim} N(E(X), \text{var}(X)) = N(m, m). \text{ Use this.}$$

Hint for ex 5:12: Use the central limit theorem (CLT) (Rice Theorem B on page 184), which says that if X_1, X_2, \dots are *iid* random variables with $E(X_i) = 0$ and $\text{var}(X_i) = \sigma^2$,

then the sum, $S_n = \sum_{i=1}^n X_i$, is approximately $N(0, n\sigma^2)$ distributed for large n . This the

same as saying that $\frac{S_n}{\sigma\sqrt{n}} \underset{\text{approximately}}{\sim} N(0, 1)$, or $P\left(\frac{S_n}{\sigma\sqrt{n}} \leq x\right) \approx \Phi(x)$, where $\Phi(x)$ is the cdf in $N(0, 1)$.