ECON 4135, APPLIED STATISTICS AND ECONOMETRICS

Initial test, 21 August 2006

Try to answer all the questions.

Your Name:

1. Let X and Y be discrete random variables that can attain the values 1, 2, 3 only, and with joint probability distribution as follows. For example, $P(X = 3, Y = 1) = 0.10$.

x \ y	1	2	3
1	0.26	0.10	0.05
2	0.24	0	0.05
3	0.10	0.10	0.10

i) Verify that the marginal distribution of Y is given by

y	1	2	3
$P(Y = y)$	0.6	0.2	0.2

ii) Find E(Y).

iii) Are X and Y independent? (Yes/no/don't know)

iv) Find $P(X = Y)$.

v) Verify that the conditional distribution of Y given $X = 3$ is given by

y	1	2	3
$P(Y = y \mid X = 3)$	0.33	0.33	0.33

vi) *The regression of Y w.r.t X* is defined as the function, $\mu(x) = E(Y \mid X=x)$, i.e. the expected value of Y in the conditional distribution of Y given that $X=x$. Complete the following table. Is the regression of Y w.r.t. X a linear function of x in this case?

x	1	2	3
$\mu(x)=E(Y \mid x)$			1.98

vii) What is the correlation, ρ , between X and Y when $\text{var}(X) = 0.70$, $\text{var}(Y) = 0.64$, and $\text{cov}(X, Y) = 0.17$? You need only calculate ρ from these moments.

viii) Let $Z = X - Y$. What is $E(Z)$ and $\text{var}(Z)$?

2. Suppose that the heights of Norwegian males of age 20 are normally distributed with expectation, 180cm, and standard deviation, 6cm, (i.e. N(180, 6^2)).

i) Find the probability that a randomly chosen 20 year old Norwegian male is taller than 190cm. (Use Table 1 in Stock and Watson (pp. 642-3) or the table given below.)

ii) Find the 10% and 90% quantiles in the distribution of male heights in Norway.

3. Let $X_1, X_2, ...,X_{36}$, be n=30 independent observations of the random variable X, which represents the height of a randomly chosen Norwegian male, regardless of age. Let X have expected value μ and standard deviation σ.

i) Draw a rough sketch of your guess of the density of X . Indicate by dotted line the normal density of the height of a randomly selected 20 year old Norwegian male.

ii) Would you expect μ to be larger or smaller than 180. On which side of 6 would you expect σ to lie?

iii) Under this model, what do you know about the statistical properties, exactly or approximately, of the sample mean, $\overline{X}$, (e.g. about its expectation, standard deviation, and distribution)?

4. Suppose α is an unknown parameter in an econometric model. Based on a large data set α was estimated by $\hat{\alpha} = 4$ with estimated standard deviation (standard error) 2. It is known that $\hat{\alpha}$ is unbiased, and approximately normally distributed in large samples.

i) Test whether α is different from 0? (Use level of significance 10%)

ii) Compute an approximate 95% confidence interval for α.

iii) Calculate the p-value when testing H_0: $\alpha \geq 7.6$ against H_1: $\alpha < 7.6$

5. Consider the following three regression models. μ denotes the expected value of Y given the explanatory variables. The main interest is to study the effect of x on Y. U is the error term with expectation 0, independent of x and z, and with $\text{var}(U) = \sigma^2$.

(a) $Y = \mu(x) + U = \alpha + \beta x + xU$

(b) $Y = \mu(x,z) + U = \beta x + \gamma z + xU$

(c) $Y = \mu(x,z) + U = \alpha + \beta_1 x + \beta_2 x^2 + \gamma_1 z + \gamma_2 z^2 + xU$

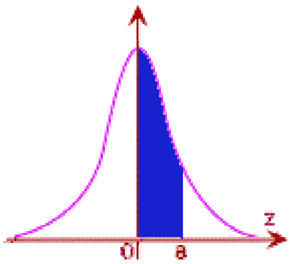
i) Interpret β in model (a) and (b).

ii) What is the ceteris paribus regression effect of x on Y in model (c)? [I.e. find the effect on $\mu = E(Y \mid x, z)$ by a unit change in x while keeping z constant.]

iii) Suppose you find model (a) to be appropriate for your data, and that the OLS (ordinary least squares) calculations yields $R^2 = 0.81$. What does this number mean?

iv) What is the conditional variance $\text{var}(Y \mid x)$? What is the variance of the transformed variable $Y^* = Y / x$? Explain why ordinary least squares regression (OLS) on this transformed variable w.r.t. $x^* = 1/x$ yields valid and optimal results. Why is still $\hat{\beta}$ estimating the effect of x on Y?

Appendix: Probabilities of the normal distribution



P(0 ≤z ≤a)

a	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852

0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990