Exam 4137 – Spring 2020

IMPORTANT: Always motivate your answers. Show knowledge and understanding of the concepts taught in the course. Your answers should be as short as possible and as long as necessary. Total marks of the examination is 170. Each subquestion in problem 1 carries 5 marks. Sub-questions in problem 2 and 3 carry 10 marks each.

1. Give brief answers to the following question.

   (a) Explain why Random Controlled Trial (RCT) is sometimes argued to be the "gold standard" for empirical causal research. Suppose that you want to study the impact of number of kids on household income for married couples in Norway. Can RCT be employed to study this particular question?

   ANSWER HINT:
   RCT relies on the random assignments of treatment status, i.e. $T_i \perp \{Y_i(1), Y_i(0)\}$. This essentially rule out all sources of selection bias and thus considered as the "gold standard". Not possible – cannot randomly assign number of kids.

   (b) Instead, you collect some information from a sample of married couples. For couple i, you observe household income y_i, the number of kids x_i and average education years for the couple z_i. One of your friends suggests to run the following regression:

   $$y_i = \alpha + \beta x_i + \gamma z_i + \varepsilon_i. \quad (1)$$

   What is the main assumption needed for the Ordinary Least Square (OLS) estimator of β to have a causal interpretation?

   ANSWER HINT:
   The CIA: $E[\varepsilon_i|x_i, z_i] = E[\varepsilon_i|z_i] = 0$

   (c) Suppose that the assumption you made in (b) holds and $cov(x_i, z_i) < 0$,what can you say about the OLS estimate from a bi-variate regression of y_i on x_i in this case?

   ANSWER HINT:
   Inconsistent, will have a negative bias.

   (d) Suppose that the variation of income for couples with more than two kids is much larger than those with two or less kids, what potential problem will this cause when estimating (1)? How will you deal with this problem?

   ANSWER HINT:
   Heteroskadasticity problem. OLS estimator is consistent but not efficient. OLS standard error formula is wrong. WLS(FGLS) or robust standard errors.

You are worried that some other variables such as religion, region and country of origin may affect both the household's decisions to have kids and its income. Unfortunately, you do not have data on these variables.

   (e) Suppose that you have information on whether the couple had twin births. Would this information help you to identify the casual effect of interest? What assumptions will you need to make? Do you think these assumptions are likely satisfied? Explain.

   ANSWER HINT:
   Yes. using twins as IV. conditions: a) relevance b) exclusion restrictions. Yes.

(f) Explain briefly in words how you would proceed to estimate the model in case (e).

ANSWER HINT:
ILS or 2SLS.

(g) Suppose instead that you have observations of household income both before and after couples having an extra kid. How will you estimate the causal effect in this case?

ANSWER HINT:
panel data (before and after analysis): can use a fixed effect model.

(h) Discuss when your method in case (g) will lead to inconsistent estimate of the causal effect of interest.

ANSWER HINT:
a) if whether to have an additional kid depend on the household income level. (feedback effects)
b) any time varying variable which correlates with the number of kids over time and have direct impact on income level, for example age. (time varying endogenous variables)

2. We want to study the relationship between hospital visits and individual characteristics. Suppose that conditional on the individual characteristics X_i, the number of hospital visits Y_i follows a Poisson distribution with parameter λ_i:

$$Prob[Y_i = y_i] = exp(-\lambda_i)\lambda_i^{y_i}/y_i!$$

where

$$\lambda_i = exp(\beta X_i).$$

Note that if $Y \sim Poisson(\lambda)$, then $f(y;\lambda) = \lambda^y e^{-\lambda}/y!$, and $E[Y] = Var(Y) = \lambda$.

We have collected information of hospital visits from 27326 individuals. We also have information on the following individual characteristics.

Edu	Years of education
Female	Female dummy (1=Female, 0=Male)
Married	Marital status (1 if married, 0 otherwise)
Age	Age
Age^2	Age square

(a) Outline in words how you would estimate β using maximum likelihood, and explain the intuition behind the method.

ANSWER HINT:
write down log-likelihood, and maximize. ie numerically iterate until first order conditions evaluated at the parameter values are zero, and second derivative is negative. the main idea is that there are no other parameters under which our data are more likely to occur.

(b) Assume that $\lambda_i = exp(\beta_0 + \beta_1 Edu_i)$. Write down the log-likelihood function excluding constant terms that do not depend on λ (i.e. the denominator $y_i!$) and derive the score function.

ANSWER HINT:
$y_i \sim Poisson(exp(\beta_0+\beta_1 Edu_i))$, its density equals $f(y_i, exp(\beta_0+\beta_1 Edu_i))$. The log-likelihood is the sum of the log of the densities, and the score the first derivative:

$$\log L = \sum_{i=1}^{N} \log f(y_i, exp(\beta_0 + \beta_1 Edu_i))$$

$$= \sum_{i=1}^{N} [y_i(\beta_0 + \beta_1 Edu_i) - exp(\beta_0 + \beta_1 Edu_i)]$$

$$\frac{\partial \log L}{\partial \beta_0} = \sum_{i=1}^{N} (y_i - exp(\beta_0 + \beta_1 Edu_i)) = 0$$

$$\frac{\partial \log L}{\partial \beta_1} = \sum_{i=1}^{N} (y_i - exp(\beta_0 + \beta_1 Edu_i))Edu_i = 0$$

(c) Use the result you obtained in (b), show that the sample mean of the estimated λ_i (that is, the estimates of λ_i when you plug in the data and the maximum likelihood estimates of the parameters) equals the sample mean of hospital visits.

> ANSWER HINT:
> the sample mean of hospital visits $\sum_{i=1}^{N} y_i$.
> Note that we also have $\hat{\lambda}_i = \sum_{i=1}^{N} exp(\hat{\beta}_0 + \hat{\beta}_1 Edu_i)$
> The FOC for β_0 is
>
> $$\frac{\partial \log L}{\partial \beta_0} = \sum_{i=1}^{N} (y_i - exp(\beta_0 + \beta_1 Edu_i)) = 0$$
>
> So it implies that $\sum_{i=1}^{N} y_i = \sum_{i=1}^{N} exp(\hat{\beta}_0 + \hat{\beta}_1 Edu_i) = \sum_{i=1}^{N} \hat{\lambda}_i$.

In the following, we assume that $\lambda_i = exp(\beta_0 + \beta_1 Edu_i + \beta_2 Female_i + \beta_3 Married_i + \beta_4 Age_i + \beta_5 Age_i^2)$.

(d) Based on the attached Stata output, test the hypothesis that the number of visits is unrelated to marriage status using the Wald test. You can use the attached critical value table.

> ANSWER HINT:
> Ward test: (-0.038)^2/(0.0397)^2=0.97^2=0.94, which has a $\chi^2(1)$ distribution under the null (1 df because 1 restriction) Not significant at 25% level.
> Or one can use the fact the p-value of the Wald test is simply the p-value from the z-test on Married ($p = 0.33$)

(e) Compute the average marginal effect of an additional year in education on the expected number of visits.

> ANSWER HINT:
> $E[y_i] = exp(\beta_0 + \beta_1 Edu_i + \beta_2 Female_i + \beta_3 Married_i + \beta_4 Age_i + \beta_5 Age_i^2)$
> $\frac{dE[y_i]}{dEdu_i} = \lambda_i \beta_1$
> So $AME = \frac{1}{N} \sum \hat{\lambda}_i \hat{\beta}_1 = \bar{y} \hat{\beta}_1 = -.0734901 * .1382566 \simeq -0.01$

(f) Carry out a likelihood ratio test of the hypothesis that the five coefficients on Edu, $Female$, $Married$, Age and Age^2 are all zero.

ANSWER HINT:
to perform the LR test under $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$ we need to maximize (and compute) the log-likelihood under the null:

$$\begin{aligned}
\log L &= \sum_{i=1}^{N} [y_i \beta_0 - exp(\beta_0)] \\
&= N(\bar{y}\beta_0 - exp(\beta_0)) \\
\partial \log L / \partial \beta_0 &= N(\bar{y} - exp(\beta_0)) = 0 \\
\hat{\beta}_0 &= log(\bar{y})
\end{aligned}$$

The LR test is $LR = -2(LogL_{restricted} - LogL_{unrestricted})$, and the restricted log likelihood equals

$$\begin{aligned}
\log L &= N(\bar{y}\beta_0 - exp(\beta_0)) \\
&= 27326 * (log(.1382566) * .1382566 - .1382566) \\
&\approx -11253.6
\end{aligned}$$

The unrestricted LL is approx.-11168.3 from the output, and the LR test statistic therefore

$$LRT \approx -2 * (-11253.6 - -11168.3) \approx 170.6$$

which has a $\chi^2(5)$ distribution under the null (5 df because 5 restriction). Which means that we reject at the 1% level since $170.6 > 15.09$.
NOTE: the exact value of the LRT may differ – some may use less precision level of $\bar{y}$. This should not matter for the final conclusion of hypothesis testing.

Stata Output

```
 . su hospvis educ female married age age2

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
     hospvis |     27,326    .1382566     .884339          0         51
        educ |     27,326    11.32063    2.324885          7         18
      female |     27,326    .4787748    .4995584          0          1
     married |     27,326    .7586182    .4279291          0          1
         age |     27,326    43.52569    11.33025         25         64
        age2 |     27,326    2022.855    1004.078        625       4096
-------------+---------------------------------------------------------


. ml model lf loglikelihoodi (hospvis = educ female married age age2)
. ml maximize

                                                Number of obs     =     27,326

Log likelihood = -11168.252
------------------------------------------------------------------------------
     hospvis |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |  -.0734901   .0084125    -8.74   0.000    -.0899782   -.0570019
      female |   .0802982   .0333027     2.41   0.016      .015026    .1455703
     married |  -.0384616   .0396869    -0.97   0.332    -.1162465    .0393234
         age |   -.013886   .0125443    -1.11   0.268    -.0384724    .0107004
        age2 |   .0002544   .0001393     1.83   0.068    -.0000185    .0005274
       _cons |  -1.089854   .2807546    -3.88   0.000    -1.640123   -.5395851
------------------------------------------------------------------------------


Critical Values of Chi-square
df       .50     .25     .10     .05     .025    .01     .001
1        0.45    1.32    2.71    3.84    5.02    6.63    10.83
2        1.39    2.77    4.61    5.99    7.38    9.21    13.82
3        2.37    4.11    6.25    7.81    9.35    11.34   16.27
4        3.36    5.39    7.78    9.49    11.14   13.28   18.47
5        4.35    6.63    9.24    11.07   12.83   15.09   20.52
6        5.35    7.84    10.64   12.59   14.45   16.81   22.46
7        6.35    9.04    12.02   14.07   16.01   18.48   24.32
8        7.34    10.22   13.36   15.51   17.53   20.09   26.12
9        8.34    11.39   14.68   16.92   19.02   21.67   27.88
10       9.34    12.55   15.99   18.31   20.48   23.21   29.59
```

3. A recent paper investigated how air temperature influences the transmission of COVID-19. The authors calculated the daily effective reproductive number, R_{it}, for 100 Chinese cities from January 21 to January 23, 2020. They also collected the average daily temperature x_{it} and humidity z_{it} for these cities during the same periods. They then run the following regression:

$$R_{it} = \alpha + \beta x_{it} + \gamma z_{it} + \varepsilon_{it} \tag{2}$$

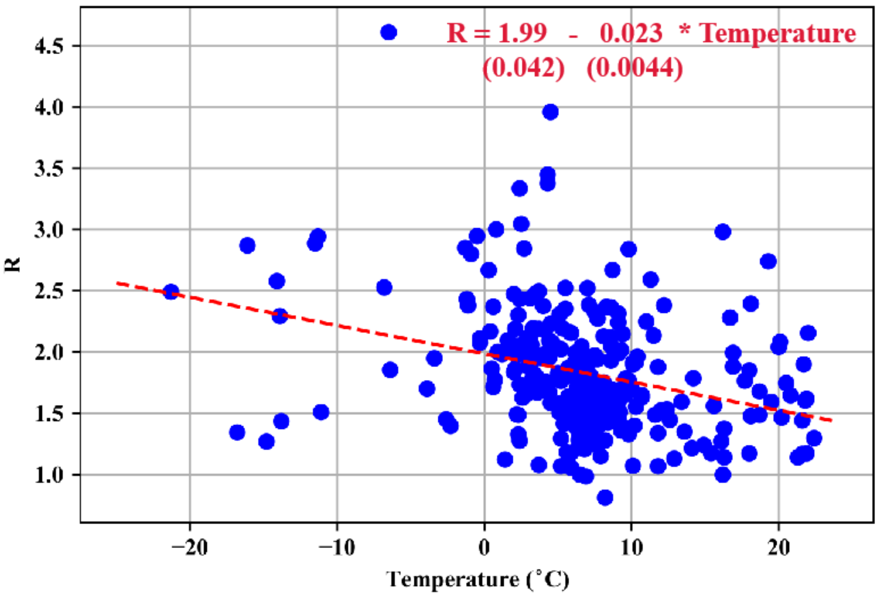
(a) What is the parameter of interest? What variation in the data did the authors use to estimate it in (2)?

ANSWER HINT:
The parameter of interest is β . The identification strategy used here is a comparison of the daily effective reproductive numbers across different temperature levels. Both the over-time variation for a given city (within) and regional differences over cities (between) are used.

(b) The authors believe that medical condition and crowdedness inside a city are important factors that influence the reproductive number R. Thus, they also included population density and the GDP per capita (both measured in year 2018) in the regression. Do you think this is necessary given their purpose?

ANSWER HINT:
It depends on whether these two variables are correlate with the variables of interest: temperature.
a)If they are independent, we can omit them from the regression and still obtain consistent estimates.
b) If they are correlated, then omit them from the regression will cause inconsistency.
Some students may think one step further: discuss this problem based on direction of the casual link: $GDP \to x_i$ or $x_i \to GDP$ or there is some unobserved factor influence both. This should be encouraged.

(c) The following diagram gives the scatter plot of R_{it} and x_{it} based on the data for the analysis. What can you learn from this diagram? Do you think the authors' conclusion that higher temperature reduce R reasonable, why?



ANSWER HINT:
The scatter plot suggests a negative relationship between R_{it} and x_{it}.
However, this result seems to be driven by the few (5-6 among total 300) observations where the R values are particularly high ($R_{it} > 3$). This makes the results less convincing.
a) Possible measure errors?
b) Something special for these cities with particularly high R?

Utilizing the panel structure of data, the authors also estimated a fixed effect and a random effect model, and the estimation results are given in the following table:

	Fixed Effects		*Random Effects*	
	Estimate	*T-statistics*	Estimate	*T-statistics*
Temperature	-0.0383	-3.27	-0.024	-3,97
Humidity	-0.0224	-10.18	-0.020	-3.06
constant	3.968	19.04	3.877	19.60
Additional controls			✓	
R^2	17%		19%	

(d) What type of variation is used to identify the parameter of interest when estimating a fixed effect model? What assumptions are needed for the estimate from the fixed effect model to be consistent?

ANSWER HINT:
Only within variation.
the strict exogeneity condition

$$E[\varepsilon_{it}|a_i, x_{i1}, x_{i2}, x_{i3}] = 0, \qquad t = 1, \ldots, 3$$

(e) Interpret the estimates for the fixed effect model, assuming that the assumptions required are satisfied.

> ANSWER HINT:
> High temperature and relative humidity reduce the transmission of COVID-19 both with 1% significance levels.
> one degree increase will reduce the R by 0.04.
> one unit increase in humidity will reduce the R by 0.02.

(f) Explain the difference between a random effect model and a fixed effect model. The authors reported that p value of a Hausman test is 0.06. Explain briefly what a Hausman test is. Based on the test result, which model would you choose?

> ANSWER HINT:
> Stronger assumptions for the random effect model.
> - fixed effect model: *the strict exogeneity* condition
> - random effect model *the strict exogeneity* condition $+E[a_i|x_{i1},\ldots,x_{iT}] = 0$
> Hausman test:
>
> - $H_0 : E[a_i|x_i] = 0$, both RE and FE estimators are consistent (but the RE estimator is more efficient)
> - $H_1 : E[a_i|x_i] \neq 0$, only the FE estimator is consistent.
>
> P value =0.06, not reject H_0 at 5% LOS but reject H_0 at 10% LOS
> The student should choose the preferred model based on her choice of LOS.

(g) The authors claimed that the results from the fixed effect model provide a way to predict the R values of a city given its temperature x and humidity z:

$$R = 3.97 - 0.038x - 0.022z$$

They then proceeded to use this equation to predict the R values for cities worldwide, using the 2019 values of March and July temperature and humidity for these cities. For example, they estimated the R value for Tokyo in July will be below 1. Suppose for now that the assumptions for the fixed effect model are satisfied. If you work for the International Olympic Committee (IOC), what suggestion would you give to IOC w.r.t the Tokyo 2020 Olympic Games? Explain your answer.

> ANSWER HINT: .
> Cannot rely on this estimate to form recommendation.
> In the fixed effect model, the city specific effects a_i is not identified. One can not use these estimates to predict the level – at best, it can be used to predict the changes in R when temperature changes.