

A forgotten piece of theory
(spring 19 before teaching-free week)

Let $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \vec{f}(x, y)$ nonlinear system

and assume x^*, y^* is a saddle point

where the Jacobian has eigenvalues $\mu > 0 > \lambda$

and λ has eigenvector $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$.

Then:

The limit of the slope of the (non-constant)
convergent paths, is $\frac{v_2}{v_1}$ (infinite if $v_1 = 0$).
↑
vertical slope.

In math:

$$\lim_{t \rightarrow +\infty} \frac{y(t) - y^*}{x(t) - x^*} = \frac{v_2}{v_1} \quad \text{for those } \checkmark \text{ (non-constant) } (x(t), y(t))$$

for which $\lim_{t \rightarrow +\infty} (x(t), y(t)) = (x^*, y^*)$.

(For linear systems: $\frac{y - y^*}{x - x^*} = \frac{v_2}{v_1}$ even w/o "lim".)