Solution key: (Questions 2 & 3)
Exercise 8.16

a) Separate least square estimation gives $\widehat{\sigma}_G^2 = 0.0170824^2 = 2.9181 \times 10^{-4}$ and $\widehat{\sigma}_A^2 = 0.0391752^2 = 1.5347 \times 10^{-3}$.

b) The critical values for testing $H_0 : \sigma_G^2 = \sigma_A^2$ against $H_1 : \sigma_G^2 \neq \sigma_A^2$ at a 5% significance level are $F_{(0.025,15,15)} = 0.349$ and $F_{(0.975,15,15)} = 2.862$. The value of the F-statistic is

$$F = \frac{\widehat{\sigma}_G^2}{\widehat{\sigma}_A^2} = \frac{2.9181 \times 10^{-4}}{1.5347 \times 10^{-3}} = 0.19014,$$

which is < 0.349. We, therefore, reject H_0 and conclude that error variances of Germany and Austria, are not the same).

c) To find the generalized least square estimate, divide all the observations that correspond to Germany by $\widehat{\sigma}_G$ and all the observations that correspond to Austria, by $\widehat{\sigma}_A$, and then run the OLS regression. The following is the regression output produced in PCGIVE (GLS coefficients are marked in red color):

Modelling LGAS/sigma by OLS-CS (using gasga.xls)
The estimation sample is: 1 to 38

	Coefficient	Std.Error	t-value	t-prob	Part.R^2
1/sigma	2.02244	0.4006	5.05	0.000	0.4285
LINC/sigma	-0.445905	0.1845	-2.42	0.021	0.1466
LPRICE/sigma	-0.297138	0.1261	-2.36	0.024	0.1404
LCARS/sigma	0.102937	0.1142	0.902	0.374	0.0233

sigma	2.54795	RSS	220.729697
log-likelihood	-87.3474	DW	0.404
no. of observations	38	no. of parameters	4
mean(LGAS/sigma)	166.518	var(LGAS/sigma)	3909.33

d) The critical value for the one-tail test $H_0 : \beta_3 \geq -1$ against $H_1 : \beta_3 < -1$ at 5% significance level is $t_{(0.05,34)} = -1.691$. The value of the t-statistic is

$$t = \frac{-0.297 - (-1)}{0.126} = 5.579,$$

which is > -1.691. We, therefore, do not reject H_0 and conclude that there is not enough evidence to suggest that demand is elastic.

Exercise 9.1

Q1) The results suggest that the regression coefficients are significant different from zero. R^2 is also very high. For testing $H_0 : \beta_1 \geq 0$ against $H_1 : \beta_1 < 0$, the test statistics $t = -7.77$, which leads to rejection of H_0 at every reasonable significance level.

Q2) Plot in output 3 indeed suggesting heteroskedasticity (point out the large variation in errors for high values of X) To calculate $t - HCSE$, you should use $HCSE$ instead of SE.

$$t - HCSE: \begin{array}{cc} \text{Constant} & \frac{18.093}{5.8031} = 3.1178 \\ \text{G} & -3.07917 \\ \text{X} & 6.80424 \end{array}$$

Q3) See the discussion on the Goldfeld-Quandt test from the book. The test statistic is given by

$$F = \frac{52.6624^2}{6.02232^2} = 76.47.$$

For the one-sided test $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 > \sigma_2^2$ (1 corresponds to the group of countries with large gross national product), the critical values at 5% significance level is given by $F_{(0.05,12,12)} = 2.687$, which is below the realized value of the F-statistic. Hence, we reject H_0 and conclude that there is evidence to suggests that $\sigma_1^2 > \sigma_2^2$.

Q4) One way to find whether the transformation has been able to reduce heteroskedasticity is to look at how "close" the standard error estimates of the regression coefficients (from OLS regression) are to the White-HCSE estimates of the same. For example, comparing $\widehat{\sigma}_{\beta_G}^{White}$ with $\widehat{\sigma}_{\beta_G}^{OLS}$, we can see that the ratio has moved close to 1 (from $\frac{0.34912}{0.13834} = 2.524$ to $\frac{0.2567}{0.1613} = 1.59$). Similarly, the ratio of $\widehat{\sigma}_{\beta_X}^{White}$ over $\widehat{\sigma}_{\beta_X}^{OLS}$ has also moved close to 1.

Q5) Recall the discussion on omitted variable bias. If r has been omitted from the model specification (as given in equation (2)) and if the true relation between r and $(G/P, X/P)$ is given by $r = \alpha_0 + \alpha_1 \frac{G}{P} + \alpha_2 \frac{X}{P}$, then the estimate of β_1 in equation (2) effectively provides an estimate of $\gamma_1 + \gamma_3\alpha_1$ (comparing the specification in (2) with the specification in (3)). Therefore, even if $\gamma_1 = 0$, we can get a significantly (statistically) negative estimate of β_1 when $\alpha_1 > 0$ and $\gamma_3 < 0$. This is precisely what the skeptics are arguing.

Q6) On the contrary, the supporters are arguing that α_1 is likely to be zero when capital is perfectly mobile across different countries. And therefore, $\beta_1 < 0$ is equivalent of having $\gamma_1 < 0$.