2.13 X is a Bernoulli random variable with $\Pr(X = 1) = 0.99$, Y is distributed $N(0, 1)$, W is distributed $N(0, 100)$, and X, Y, and W are independent. Let $S = XY + (1 - X)W$. (That is, $S = Y$ when $X = 1$, and $S = W$ when $X = 0$.)

a. Show that $E(Y^2) = 1$ and $E(W^2) = 100$.

b. Show that $E(Y^3) = 0$ and $E(W^3) = 0$. (*Hint:* What is the skewness for a symmetric distribution?)

c. Show that $E(Y^4) = 3$ and $E(W^4) = 3 \times 100^2$. (*Hint:* Use the fact that the kurtosis is 3 for a normal distribution.)

d. Derive $E(S)$, $E(S^2)$, $E(S^3)$ and $E(S^4)$. (*Hint:* Use the law of iterated expectations conditioning on $X = 0$ and $X = 1$.)

e. Derive the skewness and kurtosis for S.

2.14 In a population $\mu_Y = 100$ and $\sigma_Y^2 = 43$. Use the central limit theorem to answer the following questions:

a. In a random sample of size $n = 100$, find $\Pr(\overline{Y} \leq 101)$.

b. In a random sample of size $n = 165$, find $\Pr(\overline{Y} > 98)$.

c. In a random sample of size $n = 64$, find $\Pr(101 \leq \overline{Y} \leq 103)$.

2.15 Suppose Y_i, $i = 1, 2, \ldots, n$, are i.i.d. random variables, each distributed $N(10, 4)$.

a. Compute $\Pr(9.6 \leq \overline{Y} \leq 10.4)$ when (i) $n = 20$, (ii) $n = 100$, and (iii) $n = 1{,}000$.

b. Suppose c is a positive number. Show that $\Pr(10 - c \leq \overline{Y} \leq 10 + c)$ becomes close to 1.0 as n grows large.

c. Use your answer in (b) to argue that $\overline{Y}$ converges in probability to 10.

2.16 Y is distributed $N(5, 100)$ and you want to calculate $\Pr(Y < 3.6)$. Unfortunately, you do not have your textbook and do not have access to a normal probability table like Appendix Table 1. However, you do have your computer and a computer program that can generate i.i.d. draws from the $N(5, 100)$ distribution. Explain how you can use your computer to compute an accurate approximation for $\Pr(Y < 3.6)$.

2.17 Y_i, $i = 1, \ldots, n$, are i.i.d. Bernoulli random variables with $p = 0.4$. Let $\overline{Y}$ denote the sample mean.

a. Use the central limit to compute approximations for

i. $\Pr(\overline{Y} \geq 0.43)$ when $n = 100$.

ii. $\Pr(\overline{Y} \leq 0.37)$ when $n = 400$.