ii. Is there statistically significant evidence that students will perform better on their second attempt after taking the prep course?

iii. Students may have performed better in their second attempt because of the prep course or because they gained test-taking experience in their first attempt. Describe an experiment that would quantify these two effects.

X **3.17** Read the box "The Gender Gap of Earnings of College Graduates in the United States" in Section 3.5.

a. Construct a 95% confidence interval for the change in men's average hourly earnings between 1992 and 2008.

b. Construct a 95% confidence interval for the change in women's average hourly earnings between 1992 and 2008.

c. Construct a 95% confidence interval for the change in the gender gap in average hourly earnings between 1992 and 2008. (*Hint:* $\overline{Y}_{m,1992} - \overline{Y}_{w,1992}$ is independent of $\overline{Y}_{m,2008} - \overline{Y}_{w,2008}$.)

3.18 This exercise shows that the sample variance is an unbiased estimator of the population variance when $Y_1, \ldots, Y_n$ are i.i.d. with mean μ_Y and variance σ_Y^2.

a. Use Equation (2.31) to show that $E[(Y_i - \overline{Y})^2] = \text{var}(Y_i) - 2\text{cov}(Y_i, \overline{Y}) + \text{var}(\overline{Y})$.

b. Use Equation (2.33) to show that $\text{cov}(\overline{Y}, Y_i) = \sigma_Y^2/n$.

c. Use the results in (a) and (b) to show that $E(s_Y^2) = \sigma_Y^2$.

3.19 a. $\overline{Y}$ is an unbiased estimator of μ_Y. Is $\overline{Y}^2$ an unbiased estimator of μ_Y^2?

b. $\overline{Y}$ is a consistent estimator of μ_Y. Is $\overline{Y}^2$ a consistent estimator of μ_Y^2?

X **3.20** Suppose that (X_i, Y_i) are i.i.d. with finite fourth moments. Prove that the sample covariance is a consistent estimator of the population covariance, that is, $s_{XY} \xrightarrow{p} \sigma_{XY}$, where s_{XY} is defined in Equation (3.24). (*Hint:* Use the strategy of Appendix 3.3 and the Cauchy–Schwartz inequality.)

X **3.21** Show that the pooled standard error $[SE_{pooled}(\overline{Y}_m - \overline{Y}_w)]$ given following Equation (3.23) equals the usual standard error for the difference in means in Equation (3.19) when the two group sizes are the same $(n_m = n_w)$.