2.22 Suppose you have some money to invest—for simplicity, \$1—and you are planning to put a fraction w into a stock market mutual fund and the rest, $1 - w$, into a bond mutual fund. Suppose that \$1 invested in a stock fund yields R_s after 1 year and that \$1 invested in a bond fund yields R_b, suppose that R_s is random with mean 0.08 (8%) and standard deviation 0.07, and suppose that R_b is random with mean 0.05 (5%) and standard deviation 0.04. The correlation between R_s and R_b is 0.25. If you place a fraction w of your money in the stock fund and the rest, $1 - w$, in the bond fund, then the return on your investment is $R = wR_s + (1 - w)R_b$.

a. Suppose that $w = 0.5$. Compute the mean and standard deviation of R.

b. Suppose that $w = 0.75$. Compute the mean and standard deviation of R.

c. What value of w makes the mean of R as large as possible? What is the standard deviation of R for this value of w?

d. (Harder) What is the value of w that minimizes the standard deviation of R? (Show using a graph, algebra, or calculus.)

2.23 This exercise provides an example of a pair of random variables X and Y for which the conditional mean of Y given X depends on X but $\text{corr}(X, Y) = 0$. Let X and Z be two independently distributed standard normal random variables, and let $Y = X^2 + Z$.

a. Show that $E(Y|X) = X^2$.

b. Show that $\mu_Y = 1$.

c. Show that $E(XY) = 0$. (*Hint:* Use the fact that the odd moments of a standard normal random variable are all zero.)

d. Show that $\text{cov}(X, Y) = 0$ and thus $\text{corr}(X, Y) = 0$.

2.24 Suppose Y_i is distributed i.i.d. $N(0, \sigma^2)$ for $i = 1, 2, \ldots, n$.

a. Show that $E(Y_i^2/\sigma^2) = 1$.

b. Show that $W = (1/\sigma^2)\sum_{i=1}^{n} Y_i^2$ is distributed χ^2_n.

c. Show that $E(W) = n$. [*Hint:* Use your answer to (a).]

d. Show that $V = Y_1 \Big/ \sqrt{\dfrac{\sum_{i=2}^{n} Y_i^2}{n-1}}$ is distributed t_{n-1}.

2.25 (Review of summation notation.) Let $x_1, \ldots, x_n$ denote a sequence of numbers, $y_1, \ldots, y_n$ denote another sequence of numbers, and a, b, and c denote three constants. Show that