

- b. How large would  $n$  need to be to ensure that  $\Pr(0.39 \leq \bar{Y} \leq 0.41) \geq 0.95$ ? (Use the central limit theorem to compute an approximate answer.)
- 2.18** In any year, the weather can inflict storm damage to a home. From year to year, the damage is random. Let  $Y$  denote the dollar value of damage in any given year. Suppose that in 95% of the years  $Y = \$0$ , but in 5% of the years  $Y = \$20,000$ .
- What are the mean and standard deviation of the damage in any year?
  - Consider an “insurance pool” of 100 people whose homes are sufficiently dispersed so that, in any year, the damage to different homes can be viewed as independently distributed random variables. Let  $\bar{Y}$  denote the average damage to these 100 homes in a year. (i) What is the expected value of the average damage  $\bar{Y}$ ? (ii) What is the probability that  $\bar{Y}$  exceeds \$2000?
- 2.19** Consider two random variables  $X$  and  $Y$ . Suppose that  $Y$  takes on  $k$  values  $y_1, \dots, y_k$  and that  $X$  takes on  $l$  values  $x_1, \dots, x_l$ .
- Show that  $\Pr(Y = y_j) = \sum_{i=1}^l \Pr(Y = y_j | X = x_i) \Pr(X = x_i)$ . [Hint: Use the definition of  $\Pr(Y = y_j | X = x_i)$ .]
  - Use your answer to (a) to verify Equation (2.19).
  - Suppose that  $X$  and  $Y$  are independent. Show that  $\sigma_{XY} = 0$  and  $\text{corr}(X, Y) = 0$ .
- 2.20** Consider three random variables  $X$ ,  $Y$ , and  $Z$ . Suppose that  $Y$  takes on  $k$  values  $y_1, \dots, y_k$ , that  $X$  takes on  $l$  values  $x_1, \dots, x_l$ , and that  $Z$  takes on  $m$  values  $z_1, \dots, z_m$ . The joint probability distribution of  $X$ ,  $Y$ ,  $Z$  is  $\Pr(X = x, Y = y, Z = z)$ , and the conditional probability distribution of  $Y$  given  $X$  and  $Z$  is  $\Pr(Y = y | X = x, Z = z) = \frac{\Pr(Y = y, X = x, Z = z)}{\Pr(X = x, Z = z)}$ .
- Explain how the marginal probability that  $Y = y$  can be calculated from the joint probability distribution. [Hint: This is a generalization of Equation (2.16).]
  - Show that  $E(Y) = E[E(Y | X, Z)]$ . [Hint: This is a generalization of Equations (2.19) and (2.20).]
- 2.21**  $X$  is a random variable with moments  $E(X)$ ,  $E(X^2)$ ,  $E(X^3)$ , and so forth.
- Show  $E(X - \mu)^3 = E(X^3) - 3[E(X^2)][E(X)] + 2[E(X)]^3$ .
  - Show  $E(X - \mu)^4 = E(X^4) - 4[E(X)][E(X^3)] + 6[E(X)]^2[E(X^2)] - 3[E(X)]^4$ .