a. $\sum_{i=1}^{n} ax_i = a\sum_{i=1}^{n} x_i$

b. $\sum_{i=1}^{n} (x_i + y_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$

c. $\sum_{i=1}^{n} a = na$

d. $\sum_{i=1}^{n} (a + bx_i + cy_i)^2 = na^2 + b^2\sum_{i=1}^{n} x_i^2 + c^2\sum_{i=1}^{n} y_i^2 + 2ab\sum_{i=1}^{n} x_i + 2ac\sum_{i=1}^{n} y_i + 2bc\sum_{i=1}^{n} x_i y_i$

2.26 Suppose that $Y_1, Y_2, \ldots, Y_n$ are random variables with a common mean μ_Y, a common variance σ_Y^2, and the same correlation ρ (so that the correlation between Y_i and Y_j is equal to ρ for all pairs i and j, where $i \neq j$).

a. Show that $\mathrm{cov}(Y_i, Y_j) = \rho\sigma_Y^2$ for $i \neq j$.

b. Suppose that $n = 2$. Show that $E(\overline{Y}) = \mu_Y$ and $\mathrm{var}(\overline{Y}) = \frac{1}{2}\sigma_Y^2 + \frac{1}{2}\rho\sigma_Y^2$.

c. For $n \geq 2$, show that $E(\overline{Y}) = \mu_Y$ and $\mathrm{var}(\overline{Y}) = \sigma_Y^2/n + [(n-1)/n]\rho\sigma_Y^2$.

d. When n is very large, show that $\mathrm{var}(\overline{Y}) \approx \rho\sigma_Y^2$.

2.27 X and Z are two jointly distributed random variables. Suppose you know the value of Z, but not the value of X. Let $\widetilde{X} = E(X|Z)$ denote a guess of the value of X using the information on Z, and let $W = X - \widetilde{X}$ denote the error associated with this guess.

a. Show that $E(W) = 0$. (*Hint:* use the law of iterated expectations.)

b. Show that $E(WZ) = 0$.

c. Let $\hat{X} = g(Z)$ denote another guess of X using Z, and $V = X - \hat{X}$ denote its error. Show that $E(V^2) \geq E(W^2)$. [*Hint:* Let $h(Z) = g(Z) - E(X|Z)$, so that $V = [X - E(X|Z)] - h(Z)$. Derive $E(V^2)$.]

APPENDIX

2.1 Derivation of Results in Key Concept 2.3

This appendix derives the equations in Key Concept 2.3.

Equation (2.29) follows from the definition of the expectation.

To derive Equation (2.30), use the definition of the variance to write $\mathrm{var}(a + bY) = E\{[a + bY - E(a + bY)]^2\} = E\{[b(Y - \mu_Y)]^2\} = b^2E[(Y - \mu_Y)^2] = b^2\sigma_Y^2$.