2.5 Suppose that $Y_1, \ldots, Y_n$ are i.i.d. random variables with a $N(1, 4)$ distribution. Sketch the probability density of $\overline{Y}$ when $n = 2$. Repeat this for $n = 10$ and $n = 100$. In words, describe how the densities differ. What is the relationship between your answer and the law of large numbers?

2.6 Suppose that $Y_1, \ldots, Y_n$ are i.i.d. random variables with the probability distribution given in Figure 2.10a. You want to calculate $\Pr(\overline{Y} \leq 0.1)$. Would it be reasonable to use the normal approximation if $n = 5$? What about $n = 25$ or $n = 100$? Explain.

2.7 Y is a random variable with $\mu_Y = 0$, $\sigma_Y = 1$, skewness $= 0$, and kurtosis $= 100$. Sketch a hypothetical probability distribution of Y. Explain why n random variables drawn from this distribution might have some large outliers.

Exercises

2.1 Let Y denote the number of "heads" that occur when two coins are tossed.

a. Derive the probability distribution of Y.

b. Derive the cumulative probability distribution of Y.

c. Derive the mean and variance of Y.

2.2 Use the probability distribution given in Table 2.2 to compute (a) $E(Y)$ and $E(X)$; (b) σ_X^2 and σ_Y^2; and (c) σ_{XY} and $\text{corr}(X, Y)$.

2.3 Using the random variables X and Y from Table 2.2, consider two new random variables $W = 3 + 6X$ and $V = 20 - 7Y$. Compute (a) $E(W)$ and $E(V)$; (b) σ_W^2 and σ_V^2; and (c) σ_{WV} and $\text{corr}(W, V)$.

2.4 Suppose X is a Bernoulli random variable with $P(X = 1) = p$.

a. Show $E(X^3) = p$.

b. Show $E(X^k) = p$ for $k > 0$.

c. Suppose that $p = 0.3$. Compute the mean, variance, skewness, and kurtosis of X. (*Hint:* You might find it helpful to use the formulas given in Exercise 2.21.)

2.5 In September, Seattle's daily high temperature has a mean of 70°F and a standard deviation of 7°F. What are the mean, standard deviation, and variance in °C?

2.6 The following table gives the joint probability distribution between employment status and college graduation among those either employed or looking for work (unemployed) in the working age U.S. population for 2008.