Exam

Question 1 (10%)

Define the following terms and explain how they are related: static game, dynamic game and repeated game.

Solution: See Watson or Varian.

Question 2 (20%)

Two agents, 1 and 2, make decisions simultaneously. Each agent has two possible actions, *P* and *W*. If both choose *P*, they each get a payoff of 1; if both choose *W*, they each get a payoff of 0; if 1 chooses *P* and 2 chooses *W*, 1 gets a payoff of -1 and 2 gets a payoff of *x*; and if 1 chooses *W* and 2 chooses *P*, 1 gets a payoff of *x* and 2 gets a payoff of -1.

A.

Write the game in normal form.

Solution: payoff of 1 first, payoff of 2 second

		Action of player 2	
		P	*W*
Action of player 1	*P*	1,1	-1,*x*
	W	*x*,-1	0,0

B.

Let $x = 2$. Explain that the strategy profile with both players choosing *W* is an equilibrium in dominant strategies. Is it a Nash equilibrium?

Solution: *WW* is an equilibrium in dominant strategies and a Nash equilibrium.

C.

Let instead $x = -1$. Are there any equilibria in dominant strategies? Are there any Nash equilibria?

Solution: There are no equilibria in dominant strategies. *PP* and WW are both Nash equilibria.

Question 3 (20%)

For the remainder of the questions let $x = 2$. Suppose the above game – the "stage game" – is repeated *T* times and that *T* is finite, i.e. $T < \infty$. The two agents discount future payoffs by the factor $0 < \delta \leq 1$.

How would you solve this repeated game? What is the equilibrium solution concept? What is the solution? Does the solution depend on the values of *T* and δ?

Solution: The game may be solved by backward induction. Since the equilibrium of the stage game is unique and equal to *WW*, the solution for the repeated game is *WW* at all stages, for any *T* and δ. This is a Subgame Perfect equilibrium.

Question 4 (30%)

Suppose instead that the game is repeated infinitely many times, i.e. $T = \infty$.

A.

Explain that the solution found above when the stage game is repeated finitely many times is also a solution when the stage game is repeated infinitely many times.

Solution: Since, at each stage, playing *W* is a best response to *W*, *WW* at every stage is an equilibrium.

B.

Is there an equilibrium of the infinitely repeated game in which both agents choose *P* at every stage? How does the answer depend on the value of δ?

Solution: Consider trigger strategies by which both agents start by choosing *P* and continue doing so as long as both agents chose P at all previous stages, and choose *W* otherwise. Along the equilibrium path, when both agents choose *P*, the present value of payoffs equal $1 + \delta \cdot 1 + \delta^2 \cdot 1 \ldots = \frac{1}{1-\delta}$. If an agent chooses *W* in a period in which the other agent chooses *P*, the present value of payoffs equal 2 (at all subsequent stages, payoffs are 0). Playing *P* is an equilibrium if $\frac{1}{1-\delta} > 2$, i.e. if $\delta > \frac{1}{2}$. There are other solutions also, with different formulations of the "punishment" phase; these require higher values of δ.

Question 5 (10%)

Consider again a repeated game, but assume now that, after any given stage, with probability π the stage game is repeated again, while with probability $1 - \pi$ the stage game is not repeated and the game stops.

Explain that this game corresponds to a game in which the stage game is repeated infinitely many times and agents' discount factor is $\pi\delta$. Discuss how the existence of an equilibrium in which both agents choose *P* at every stage depends on π.

Solution: The present value of payoffs along the equilibrium path is $1 + \pi\delta \cdot 1 + (\pi\delta)^2 \cdot 1 \ldots = \frac{1}{1-\pi\delta}$ while the gain from deviation is still 2. The equilibrium requires $\frac{1}{1-\pi\delta} > 2$, i.e. $\pi > \frac{1}{2\delta}$.

Question 6 (10%)

When studying a real-world phenomenon, the theorist should make assumptions that reflect aspects of the world that are central to phenomenon under study.

Based on the above analysis, discuss when it makes sense to assume finitely and infinitely repeated interaction between agents, respectively.

Solution: Finite repetition makes sense when there is an actual end date to the interaction, and when future interaction is infrequent (δ low) or unlikely (π low), in which case collusive equilibria do not exist. Infinite repetition makes sense when interaction is frequent and likely to continue, in which case collusive equilibria may be sustained.