

Extra exam

Question 1 (10%)

Define and explain the following terms: static game, repeated game, Nash equilibrium and subgame perfect equilibrium.

Solution: In a static game, players make all their decisions before observing what other players do; an example is the Prisoners' Dilemma. A repeated game has a static game repeated finitely or infinitely many times; an example is the Prisoners' Dilemma repeated. In a Nash equilibrium all players' strategies are best responses to opponents' strategies. In a subgame perfect equilibrium, strategies constitute a Nash equilibrium in every subgame.

Question 2 (20 %)

Consider a market for a homogenous good where (inverse) demand is given by

$$p = A - Q,$$

where p is the market price, Q is total quantity traded and A is a constant. Two firms, 1 and 2, compete for customers by setting supply simultaneously, where q_i is the supply of firm i , $i = 1, 2$. Firms incur no cost of supplying the good.

Derive the best-response curves for the two firms and explain why quantities are strategic substitutes (i.e. best-response curves are downward sloping). Demonstrate that at Nash equilibrium profits are

$$\pi_1^N = \pi_2^N = \frac{1}{9}A^2.$$

Solution: Profits may be written $\pi_i = (A - q_1 - q_2)q_i$. From the first-order conditions, it follows that best response curves are $q_i(q_j) = \frac{1}{2}(A - q_j)$, $i, j = 1, 2, j \neq i$. Quantities are strategic substitutes because a higher supply from the competitor depresses market price and makes it optimal to respond by reducing supply. Solving for equilibrium quantities yield $q_1^N = q_2^N = \frac{1}{3}A$, from which the profit expressions follow directly.

Question 3 (30 %)

Suppose time is divided into infinitely many periods of equal length and that the static game is played once in each period.

A

Do playing the equilibrium strategies of the static game in every period constitute a Nash equilibrium in the repeated game? Is this a subgame perfect equilibrium?

Solution: It is both a Nash equilibrium and a subgame perfect equilibrium; since the strategy profile constitutes a Nash equilibrium of the original game, it does so in every subgame, which are all identical to the original game.

B

The two firms discount payoffs one period ahead by the discount factor δ , with $0 \leq \delta \leq 1$. Explain that if δ is sufficiently large there exists a subgame perfect equilibrium in which the two firms each supply half the monopoly quantity, leading to per period profits of

$$\pi_1^M = \pi_2^M = \frac{1}{8}A^2,$$

and describe the (grim) equilibrium (trigger) strategy profile that supports this outcome. Explain that for the equilibrium to exist δ must satisfy the condition,

$$\pi_i^B - \pi_i^M \leq \frac{\delta}{1 - \delta} (\pi_i^M - \pi_i^N),$$

where $\pi_i^B = \frac{9}{64}A^2$ is the maximum profit from deviating from half the monopoly output. What is the critical value of δ ?

Solution: Monopoly output is $\frac{1}{2}A$. The grim trigger strategy profile is where firm i , $i = 1, 2$, play {start by setting $q_i = \frac{1}{4}A$ and continue doing so so long as both firms chose this quantity in the past; otherwise set $q_i = \frac{1}{3}A$ }. This constitutes a subgame perfect equilibrium if

$$\frac{1}{1 - \delta} \pi_i^M \geq \pi_i^B + \frac{\delta}{1 - \delta} \pi_i^N,$$

where the left-hand side is the discounted profits when both players play according to the equilibrium strategy and the right-hand side is the maximum discounted profits from deviating in the first period and getting $\pi_i^B = \frac{9}{64}A^2$ (by playing $\frac{3}{8}A$, the best response to $\frac{1}{4}A$), and subsequently playing the static Nash equilibrium strategies. Rearrangement yields the expression above, where the left-hand side may be read as the gain from deviation in the first period while the right-hand side is the loss from playing the Nash outcome rather than the monopoly outcome for the remainder of the game. The condition may be further developed to $\delta \geq \frac{9}{17}$.

Question 4 (30 %)

Suppose next that, in any individual period, the constant term A in the demand function may, with equal probability, take on either the value $A_- = 1 - \Delta$ (low demand) or the value $A_+ = 1 + \Delta$ (high demand), where $\Delta < 1$. Firms observe the value of A at the beginning of the period before making their supply decisions.

A

Consider any given period and explain that *ex ante* (i.e. before the actual value of A is realised) expected profits when the firms play the static Nash equilibrium are

$$E\pi_1^N = E\pi_2^N = \frac{1}{9}(1 + \Delta^2),$$

while when they each supply half the monopoly quantity profits are

$$E\pi_1^M = E\pi_2^M = \frac{1}{8}(1 + \Delta^2).$$

Solution: Using the result from Question 2, the expected profit of firm i at the Nash equilibrium becomes

$$E\pi_i^N = \frac{1}{2} \frac{1}{9} (1 - \Delta)^2 + \frac{1}{2} \frac{1}{9} (1 + \Delta)^2,$$

which reduces to the expression above. Profits when firms play the monopoly outcome are derived analogously.

B

Consider again the possibility of a subgame perfect equilibrium in which each firm supplies half the monopoly quantity in each period. Explain why the incentive to deviate is stronger when demand is high than when it is low. What is the condition on the discount factor for such an equilibrium to exist?

Solution: The one-period gain from deviating when demand is low equals $\pi_{i-}^B - \pi_{i-}^M = \frac{9}{64} (1 - \Delta)^2 - \frac{1}{8} (1 - \Delta)^2 = \frac{1}{64} (1 - \Delta)^2$, while the corresponding gain when demand is high equals $\pi_{i+}^B - \pi_{i+}^M = \frac{1}{64} (1 + \Delta)^2$; clearly, the latter is larger, so the incentive to deviate is stronger when demand is high. It follows that the necessary condition on the discount factor becomes

$$\pi_{i+}^B - \pi_{i+}^M \leq \frac{\delta}{1 - \delta} (E\pi_i^M - E\pi_i^N).$$

This may be further developed to

$$\delta \geq \left(1 + \frac{64}{72} \frac{1 + \Delta^2}{(1 + \Delta)^2} \right)^{-1} > \frac{9}{17}.$$

Question 5 (10 %)

Based on the analysis above, discuss how varying market conditions may affect the robustness of (tacit or explicit) collusion among firms.

Solution: It is more difficult to satisfy the necessary conditions for a collusive equilibrium when demand fluctuates than when it is stable, because the incentive to deviate increases with the level of demand.