

Exam

Question 1 (10%)

Define and explain the following terms: risk, risk aversion, beliefs and Perfect Bayesian equilibrium.

Solution: Risk is being exposed to alternative outcomes that are valued differently, such as the (stochastic) return on a financial asset. Risk aversion is valuing the expected value of a risky outcome higher than the risky outcome itself, such as the expected rate of return as opposed to the stochastic return on an asset. Beliefs refer to conjectures of actions of others or the result of one's own or others' choices, such as how an opponent in a certain game will act. A Perfect Bayesian equilibrium requires that each player's strategies specifies optimal actions, given his beliefs and the strategies of other players, and the beliefs are consistent with Bayes' rule wherever possible (cf. Watson, ch. 28). Providing examples to illustrate the concepts counts positively.

Question 2 (30%)

An entrepreneur has developed a potential project. If the entrepreneur invests an amount $x \geq 0$ in the project it gives a return $\bar{r}x$ with probability π and a return $\underline{r}x$ with probability $1 - \pi$, where $\underline{r} < 0 < \bar{r}$.

A

Assume that the entrepreneur values monetary gains y according to a twice continuously differentiable, increasing and concave utility function $v(y)$, with $v'(y) > 0$ and $v''(y) < 0$. Explain why this implies that the entrepreneur is risk averse and that the entrepreneur therefore prefers the expected value of the return on the investment, $v((\pi\bar{r} + (1 - \pi)\underline{r})x)$, to the expected utility of the project, $\pi v(\bar{r}x) + (1 - \pi)v(\underline{r}x)$. Illustrate your argument in a figure.

Solution: Risk aversion follows from the definition of concavity, specifically that the utility of the expected return exceeds the expected utility of the project, cf. Varian & Melitz Chapter 12.5, in particular Figure 12.2.

B

Define $Eu(x) = \pi v(\bar{r}x) + (1 - \pi)v(\underline{r}x)$ and find optimality conditions for the level of investment, x^* , that maximises the entrepreneur's expected utility $Eu(x)$. Demonstrate that x^* is increasing in π .

Solution: The first-order and second-order conditions for optimal investment x^* may be written $Eu'(x^*) = \pi v'(\bar{r}x^*)\bar{r} + (1 - \pi)v'(\underline{r}x^*)\underline{r} = 0$ and $Eu''(x^*) = \pi v''(\bar{r}x^*)\bar{r}^2 + (1 - \pi)v''(\underline{r}x^*)\underline{r}^2 \leq 0$. The second-order condition is satisfied by the concavity of v . We may find the impact of π on x^* by implicit derivation of the first-order condition, which leads to $\frac{dx^*}{d\pi} = \frac{v'(\bar{r}x^*)\bar{r} - v'(\underline{r}x^*)\underline{r}}{-Eu''(x^*)} > 0$ (remember that $\underline{r} < 0$).

Question 3 (40%)

Suppose that, in order for the project described in Question 2 above to be realised, the entrepreneur requires support from an outside investor (the investor finances the remainder of the project, over and above what the entrepreneur puts in, x). If the project does not get support, the entrepreneur receives expected utility corresponding to an investment of zero, i.e. $x = 0$, $Eu(0) = v(0)$. The investor will support the project if and only if the likelihood of success is sufficiently high, i.e. $\tilde{\pi} > \pi^V$, where $\tilde{\pi}$ is the likelihood with which the investor believes the project will be a success and π^V is the threshold level for success.

Suppose that the project may be of one of two types, with high (H) and low (L) likelihood of success, respectively, i.e. $\pi_L < \pi_H$. We assume $\pi_L < \pi^V < \pi_H$.

A

Suppose both the entrepreneur and the investor know the type of the project. Demonstrate that, at equilibrium, the investor supports the project if and only if $\pi = \pi_H$ and that $x_L = 0$ and $x_H = x_H^*$, where, for $i = L, H$, x_i is the equilibrium investment choice of the entrepreneur if the project is of type i , and x_i^* is the investment that maximises expected utility derived in Question 2.B above when $\pi = \pi_i$.

Solution: It should be demonstrated that the suggested strategies do constitute an equilibrium. Specifically, since the investor does not support the project when the likelihood of success is low, the entrepreneur is better off not investing.

B

Suppose next that only the entrepreneur knows the type of the project. The investor holds the prior belief that the project has a high likelihood of success (i.e. $\pi = \pi_H$) with probability α and a low likelihood of success (i.e. $\pi = \pi_L$) with probability $1 - \alpha$. Before the investor decides whether to support the project, the entrepreneur invests, i.e. chooses x . We denote by β the investor's posterior belief, having observed x , that the project has a high likelihood of success, while $1 - \beta$ denotes the investor's posterior belief that the project has low likelihood of success.

Let x_L^0 be the level of investment at which the entrepreneur is indifferent between investing and not investing when $\pi = \pi_L$, i.e. $Eu_L(x_L^0) = \pi_L v(\bar{r}x_L^0) + (1 - \pi_L)v(\underline{r}x_L^0) = Eu_L(0) = v(0)$.

We first consider the case that $x_L^0 > x_H^*$.

Explain that there exists a separating Perfect Bayesian equilibrium of the game in which the entrepreneur chooses $x_L = 0$ and $x_H = x_L^0$, and the investor believes $\beta = 0$ if $x < x_L^0$ and $\beta = 1$ if $x \geq x_L^0$ and invests only if $\beta = 1$.

Solution: The entrepreneur's choice of $x_L = 0$ when $\pi = \pi_L$ is optimal given that the project will only be undertaken if $x \geq x_L^0$, a level of investment at which the entrepreneur cannot earn more than $v(0)$. The choice of $x_H = x_L^0$ when $\pi = \pi_H$ is also optimal given that, for any lower level of investment (specifically, x_H^*), the project will not be realised, at $x_H = x_L^0$ the entrepreneur obtains expected utility greater than $v(0)$ (the level of expected utility that would obtain from this investment when the likelihood of success is low), while any higher level of investment leads to lower expected utility (given that the expected utility function is decreasing for investment above x_H^*). The updating of beliefs is according to Bayes' rule for

$x_L = 0$ and $x_H = x_L^0$, and may be freely chosen for out-of-equilibrium moves. The investor's decision on supporting the project is optimal given beliefs.

C

Consider next the case that $x_L^0 < x_H^*$. Construct a pooling Perfect Bayesian equilibrium at which $x_L = x_H = x_H^*$. Explain why the existence of such an equilibrium requires that the investor expects that the project is a priori worth supporting, i.e. $\alpha\pi_H + (1 - \alpha)\pi_L > \pi^V$.

Solution: Since the investment is the same for both types of projects, the investor learns nothing from observing investment, and hence posterior beliefs equal prior beliefs, i.e., $\beta = \alpha$. It follows that the investor will support the project at equilibrium. Given that investor supports the project, when $\pi = \pi_H$ the entrepreneur can do no better than invest x_H^* . When $\pi = \pi_L$, the entrepreneur is willing to invest x_H^* if lower investment levels would lead the investor not to support the project. This would be the case if the investor held beliefs that $\beta = 0$ if $x < x_H^*$. To fully describe the beliefs, we may, for example, assume that $\beta = 1$ if $x > x_H^*$.

Question 4 (20%)

Explain how (costly) signalling may overcome problems of asymmetric information in markets for venture capital, like in the example in Question 3 above. Referring to the situation described in Question 3, under what conditions are such signalling likely to occur? Mention examples of other markets where similar signalling behaviour may occur.

Solution: Signalling reveals information when the equilibrium is separating. In the example, this happens when it is costly for the owner of project with low likelihood of success to mimic a project with high likelihood of success. This may be due, for example, to large differences in the likelihood of success ($\pi_H - \pi_L$) or large disutility of risk (concavity of v). Note also that only separating equilibria exists when there is relatively little probability that projects have a high likelihood of success (α is small), since then $\alpha\pi_H + (1 - \alpha)\pi_L < \pi^V$. Examples of other markets where signalling may occur are markets for labour (education), used cars (warranties) and insurance (deductibles).