Guide for grading.

1a. α is related to the discount factor and measures impatience.

1b. This are demands of a standard Cobb-Douglas utility function. $D_1 = \alpha m$ and $D_2 = (1-\alpha)\, m\frac{1}{r}$. The money available to Anastasia is $m = 1$ (price of good 1 is normalized to 1).

1c. Income and substitution effects perfectly cancel each other. Thus, an increase in the price of x_2 does not affect the demand of x_1.

1d. $V = \alpha \ln \alpha m + (1-\alpha) \ln (1-\alpha)\, m r^{-1}$. By Roy's identity:

$$D_2 = -\frac{V_r}{V_m} = -\frac{-\frac{1-\alpha}{r}}{\frac{\alpha}{m} + \frac{1-\alpha}{m}} = (1-\alpha)\, m \frac{1}{r}.$$

2a. The elasticity of substitution is $\frac{1}{2}$.

2b. The cost function is pretty standard. $C = \left(w_1^{\frac{1}{2}} + w_2^{\frac{1}{2}}\right)^2 y$.

2c. The cost function is linear. Thus, 3 cases can emerge: the output price is too low and optimal production is 0, profit is 0. The output price is high and optimal production is ∞, profit is ∞. The output price is $p = \left(w_1^{\frac{1}{2}} + w_2^{\frac{1}{2}}\right)^2$ and any level of production is indifferent for the firm and gives a 0 profit. However, the profit function is not well-defined.

3a. when $r = 2$, from exercise 1, we can compute $x_1^A = \alpha$ and $x_2^A = (1-\alpha)\, \frac{1}{2}$.

Colin wishes to equalize δx_1 and x_2. Since the budget constraint is $x_1 + \frac{1}{r} x_2 \leq 2$, $x_1 + \frac{1}{r}\delta x_1 = 2$ gives $x_1^C = 2\frac{r}{r+\delta}$, while $x_2^C = \delta x_1^C = 2\delta \frac{r}{r+\delta}$. When $r = 2$, $x_1^C = \frac{4}{2+\delta}$ and $x_2^C = \frac{4\delta}{2+\delta}$.

3b. The sum of the excess demands is given by the sum of the demands, since they do now own any resource of good 2. This is given by

$$x_2^A + x_2^C = (1-\alpha)\, \frac{1}{r} + 2\frac{r}{r+\delta},$$

and is positive for any positive price r. This happens because excess demands for the economy should include the production function: at the equilibrium the investment technology provides the quantities of consumption in period 2 demanded by the agents.

3c. The first welfare theorem tells that any Walrasian equilibrium is efficient. This is also the case for this economy: it is not possible to improve the well-being of one agent without making another agent worse off.

Problems in game theory and the economics of information (total weight 50%)

Guide for grading

Problem 3 (weight 10%) (Problem one of part II).

True or false? For each of the statements, if true, try to explain why, and if false, provide a counter-example.

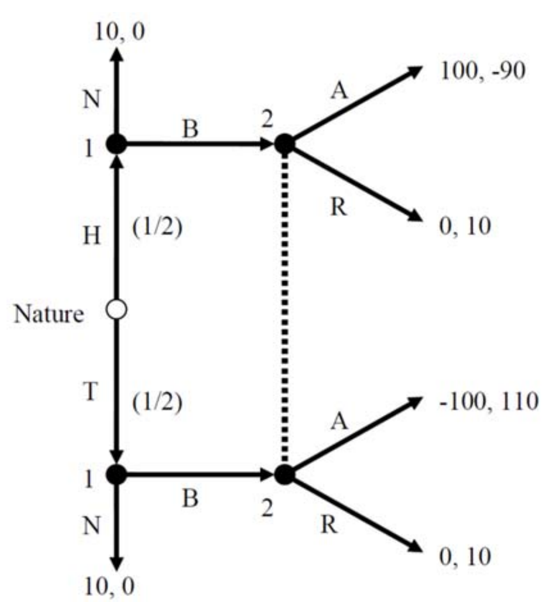
a) In a finite normal-form game, every strategy of each player is either strictly dominated or strictly dominates any other strategy.
FALSE. E.g., consider matching pennies.

b) In a finite normal-form game, if a pure strategy of a player is strictly dominated by another pure strategy, then the former strategy cannot be a best response to any belief that the player has about the strategies played by his opponents.
TRUE. For any belief, the dominating strategy is a better reply.

Problem 4 (weight 20%) (Problem two of part II).

In the presence of Mary, Peter tosses a 10-kroner coin. After seeing whether the outcome is Heads or Tails – but before showing this to Mary – Peter has the opportunity to propose a bet, which Mary must accept or reject before she sees the outcome of the toss. If he proposes the bet and Mary accepts, then Mary must pay 100 kroner to Peter if the outcome is Heads, while Peter must pay 100 kroner to Mary if the outcome is Tails. If he proposes the bet and Mary rejects, then no payments are made. Peter keeps the 10-kroner coin if he does not propose the bet, while Mary keeps the 10-kroner coin if Peter proposes the bet (independently of whether she accepts or rejects the bet and independently of the outcome of the toss).

a) Present the above situation as an extensive-form game.



b) Show that Peter never proposes the bet!

In any pure-strategy perfect Bayesian equibrium, both types of player 1 choose N and 2 chooses R. 2 assigns at least 0.5 probability on H if 1 chooses B. (There is also a set of mixed-strategy perfect Bayesian equilibria where both types of player 1 choose N, 2 chooses R with at least 0.9 probability, and 2 assigns exactly 0.5 probability on H if 1 chooses B.) Credit should be given to students who observe that player 1 of type T can only lose by choosing B.

Problem 5 (weight 20%) (Problem three of part II).

Consider an insurance market where insurance is offered by a large number of risk-neutral insurance companies and where consumers have private information about their accident probabilities, which are either low or high. Consumers are risk-averse and maximize expected utility. Their expected utility is given by

$$EU = \pi u(w - L) + (1 - \pi)u(w)$$

if they do not buy insurance, where $u' > 0$ and $u'' < 0$, w is initial wealth, L is the loss incurred in case of an accident, $\pi = \underline{\pi}$ if the accident probability is low, $\pi = \overline{\pi}$ if the accident probability is high, and $0 < \underline{\pi} < \overline{\pi} < 1$. An insurance contract specifies that a consumer pays a premium p_1 to her insurer and receives a benefit B from her insurer in case of an accident. Hence, the expected utility of contract (B, p) equals:

$$EU = \pi u(w - L + B - p) + (1 - \pi)u(w - p)$$

We should interpret $L - B$ as the deductible.

Provide short answers to the following questions:

a) Assume that there exists an equilibrium in such an insurance market. Why do the consumers with the low accident probability pay a deductible, while the consumers with the high accident probability do not?
Low-probability consumers pay a deductible to avoid that high-probability consumers choose their contract with a lower premium.

b) If you were faced with a hypothetical choice in this market, between being a consumer with the high accident probability and consuming its contract without a deductible and being a consumer with the low accident probability and consuming its contract with a deductible, which kind of consumer would you choose to be and why?
On the one, high-probability consumers are indifferent between the two contracts. On the other hand, the contract for low-probability consumers with a deductible is better for the low-probability consumers than for the high-probability consumers. It follows that one would to choose to be a low-probability consumer, if faced with such a hypothetical choice.

c) Why may an equilibrium fail to exist if there is a high proportion of consumers with the low accident probability?
A "rebel" firm could earn positive expected profits by offering a pooling contract that attracts all consumers.