Exam 3220-4220 Fall 2022

Problem 1 (Microeconomic Theory)

Part A Consumer Theory

Weight 25%

A central result in consumer theory is that the marginal rate of substitution between two goods *i* and *j* equals their relative price:

$$MRS_{ij} = \frac{p_i}{p_j}.$$

Explain the assumptions on preferences, prices and goods underlying this result, how the result may be derived and how it may be interpreted in economic terms.

Solution sketch: Students should discuss assumptions on preferences that allows for a "well-behaved" utility representation of preferences. They should then explain that the result follows from utility maximisation (it is a plus, but not warranted, that they actually derive the result formally; it is also a plus if they illustrate the solution in a figure). The interpretation should include the fact that (subjective) willingness to pay for good i in amounts of good j equals the market value of good i in amounts of good j.

Part B General Equilibrium

Weight 25%

Consider an exchange economy with a given number of consumers and goods. Explain how such an economy may be represented and how equilibrium may be characterised. Discuss the economic interpretation of equilibrium conditions.

Solution sketch: Students should explain that the economy may be represented by a set of goods and a set of consumers with their preferences over and endowments of goods. They should then explain that equilibrium requires that demand for each good equals the total endowment of that good, and that for all pairs of consumers and goods marginal rates of substitutions are equal and equal to the relative price of these goods (it is a plus if they explain how one may guarantee an interior solution; it is also a plus if the solution is illustrated in a figure for the two-goods, two-consumer case). They should explain that equilibrium prices not only ensures equilibrium but also that marginal willingness to pay is equalised across consumers and hence all gains from trade are exhausted.

Solution sketches for
the exam in ECON3220/4220, Fall 2022

Problem 2 (Game theory and the economics of information)

Part A: Rationalizable strategies and mixed-strategy Nash equilibrium

Weight: 18% *(with equal weight* = 9% *on each subproblem)*

Consider the following normal form game.

	L	C	R
U	3, 1	5, 0	2, 3
M	2, 2	1, 4	6, 1
D	5, 0	2, 1	3, 1

(a) Show that U and M are rationalizable for player 1, and C and R are rationalizable for player 2.

A strategy is rationalizable if and only if it survives iterated elimination of strictly dominated strategies. L is strictly dominated by a mixture of C and R where the probabilities of each of these strategies exceeds $\frac{1}{3}$. With L eliminated, D is strictly dominated by a mixture of U and M where the probabilities of each of these strategies exceeds $\frac{1}{4}$. No further elimination is possible. Hence, U and M are rationalizable for player 1, and C and R are rationalizable for player 2.

(b) The game has a unique Nash equilibrium, in mixed strategies. Find the mixed strategies that the players use in this Nash equilibrium.

By determining, for each player, the best response to pure strategies for the opponent, it can easily be checked that there is no Nash equilibrium in pure strategies. Since L is strictly dominated, this strategy cannot be a best response to any belief concerning the choice of player 1 and cannot be assigned positive probability in a mixed-strategy Nash equilibrium. If L is played with probability 0*, then D cannot be a best response to any remaining belief concerning the choice of player 2 and cannot be assigned positive probability in a mixed strategy Nash equilibrium. For player 1 to be indifferent between U and M, player 2 must assign probability $\frac{1}{2}$ to C and probability $\frac{1}{2}$ to R. For player 2 to be indifferent between C and R, player 1 must assign probability $\frac{1}{2}$ to U and probability $\frac{1}{2}$ to M. Hence, the mixed strategy of player 1 is $(\frac{1}{2}, \frac{1}{2}, 0)$ and the mixed strategy of player 2 is $(0, \frac{1}{2}, \frac{1}{2})$.*

Part B: Auction under incomplete information

Weight: 12%

Consider a private value first-price sealed-bid auction with two bidders who both are risk-neutral, have valuations that are identically and independently distributed on the interval $[0, 1]$ and can submit bids in the interval $[0, 1]$. In a private value first-price sealed-bid auction the bidders are first informed of their valuations, they then submit their bids, the item is assigned to the bidder with the highest bid (and with equal probability to each if the two bids are equal), and the winner pays the highest bid (i.e., his own bid). The utility of the winner equals his/her valuation minus the payment, while the other bidder has zero utility.

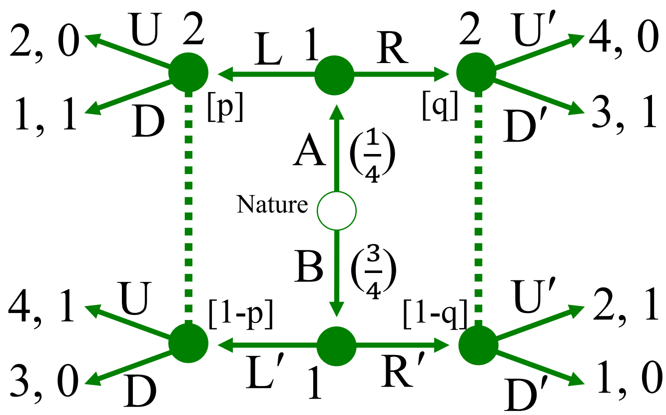
Explain why bidders bid lower than their valuation. Use the expression in Jehle and Reny Theorem 9.1 to argue that bidders in a symmetric equilibrium bid the expected valuation of the other bidder conditional on winning. What does this imply in the case where valuations are uniformly distributed? Why does this result indicate that revenues for the seller is the same as in the second-price sealed-bid auction, where the item is assigned to the bidder with the highest bid (and with equal probability to each if the two bids are equal), and the winner pays the second highest bid.

As shown in the lecture notes "Incomplete information: Bayesian Nash equilibrium" and "Auctions and revenue equivalence", bidders must bid less than their valuation in order to gain if they win the auction, and that this must be traded off against the lower probability of winning. The the expression in Jehle and Reny Theorem 9.1 is interpreted as indicated in the lecture notes on "Auctions and revenue equivalence". With a uniform distribution, bidders will exactly half their valuation, as this is the expected valuation of the other bidder conditional on winning (when both bidders use the same bidding function). In the symmetric equilibrium of a second-price auction, bidders bid their valuations but pay the bid of the other bidder when winning. In expectations, this gives the seller the same revenue.

Part C: Perfect Bayesian equilibrium

Weight: 20% *(with equal weight* = 10% *on each subproblem)*

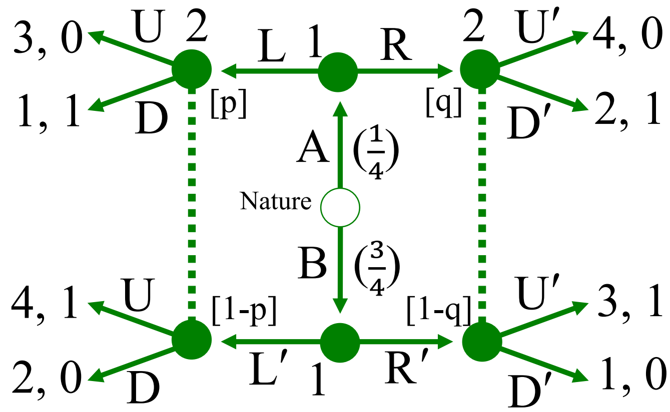
(a) Consider the following extensive form game, where the first number is the payoff of player 1 and the second number is the payoff of player 2. Notice that the probability that player 1 is of type A is $\frac{1}{4}$, and the probability that player 1 is of type B is $\frac{3}{4}$.



Show that this game has a separating Perfect Bayesian equilibrium. Does the game has any other Perfect Bayesian equilibrium?

Player 1 of type A always prefers to choose R and player of type B always prefers to choose L'. By Bayes' rule, $p = 0$ and $q = 1$. Thus, the best response for player 2 is to choose U and D'. Then it is in fact optimal for player 1 of type B to choose L' as 3 *is larger than* 2. *Therefore,* $(RL', UD'), (p = 0, q = 1)$ *is a separating Perfect Bayesian equilibrium. Since player 1 of type A always prefers to choose R and player of type B always prefers to choose L', there exists no other Perfect Bayesian equilibrium*

(b) Consider the following variant of the extensive form game under (a), where the payoff of player 1 has been changed somewhat, but where the game otherwise is unchanged.



Show that this game has a pooling Perfect Bayesian equilibrium. Does the game has any other Perfect Bayesian equilibrium?

There are two candidates for a pooling Perfect Bayesian equilibrium, one where both types of player 1 choose L/L' *and where both types of player 1 choose* R/R'. *They are in fact both Perfect Baysian equilibria:* $(LL', UD'), (p = \frac{1}{4}, q \geq \frac{1}{2})$ *and* $(RR', DU'), (p \geq \frac{1}{2}, q = \frac{1}{4})$. *A very good student will realize that this is the Beer-Quiche game and that the former of these equilibria is more plausible, since choosing L is equilibrium dominated for type A of player 1 in the second equilibrium and thus believing that* $p \geq \frac{1}{2}$ *might be considered unreasonable. There is no separating equilibrium as type A of player 1 will always want to mimic type B of player 1.*