# Postponed exam ECON3220/4220 Fall 2022 

## Problem 1 (Microeconomic Theory)

Part A
Weight 25\%
Consider an economy consisting of one firm and one consumer. The firm produces two different products with a single resource. The consumer values both products. The amount of the resource is given.

Discuss assumptions, including on firm production technology and consumer preferences, that are warranted for the existence of a general equilibrium in which both products are traded.

Solution sketch: Students should explain that there must be non-increasing returns in the production of each product, and that preferences satisfy assumptions that allow for a "wellbehaved" utility representation. In addtion, they should explain that the consumer, as owner, is entitled to the value of the resource, as well as any profits from the firm, that both the firm and the consumer and price takers, and that markets must clear. It is a plus if they set up the model formally and illustrate assumptions on production technology and preferences.

## Part B

Weight 25\%
Derive the conditions characterising the general equilibrium, including the condition

$$
M R S_{12}=\frac{p_{1}}{p_{2}}=M R T_{12}
$$

where $M R S_{12}$ and $M R T_{12}$ are the marginal rate of substitution and the marginal rate of transformation, respectively, between products 1 and 2 , and $p_{1}$ and $p_{2}$ are the prices of the two products. Interpret the results in economic terms.

Solution sketch: Students should solve the profit and utility maximisation problems of the firm and the consumer, respectively. In addition, they should present the market clearing conditions (products and resource), as well as the budget constraint with the income of the consumer (resource income and firm profits). It is a plus if equilibrium is illustrated in a figure. The interpretation should include the fact that (subjective) willingness to pay for good 1 in amounts of good 2 equals the market value of good 1 in amounts of good 2, which again equals the transformation rate in production.

## Solution sketches for the postponed exam in ECON3220/4220, Fall 2022

## Problem 2 (Game theory and the economics of information)

This problem relates to experiments made during the lectures of ECON3220/4220. Solving the different parts of the problem does not require that the student has participated in the experiments.

## Part A: Bargaining

Weight: $14 \%$ (with equal weight $=7 \%$ on each subproblem)
(i) Ultimatum bargaining. Consider an ultimatum game where player 1 can demand $0,1,2,3$, $4,5,6,7,8,9$, or 10 . If player 2 accepts, then player 1 receives his demand, while player 2 receives 10 minus the demand of player 1 . If player 2 rejects, then both players receive 0 . Show that it is a subgame-perfect Nash equilibrium outcome that player 1 demands 9 and that player 2 accepts! Are there any other subgame-perfect Nash equilibrium outcomes?

It is a subgame-perfect equilibrium strategy for player 2 to accept receiving any positive amount, while rejecting if player 1 demands 10. Demanding 9 is a best response for player 1. It is also a subgame-perfect equilibrium strategy for player 2 to always accept. Demanding 10 is a best response for player 1 in this case.
(ii) Two-period alternating offer bargaining. Consider a two-period alternating offer bargaining game where player 1 in the first period can demand $0,1,2,3,4,5,6,7,8,9$, or 10 . If player 2 accepts player 1's demand, then player 1 receives his demand, while player 2 receives 10 minus the demand of player 1. If player 2 rejects, player 2 in the second period can demand $0,1,2,3,4,5,6,7,8$, or 9 . If player 1 accepts player 2 's demand, then player 2 receives his demand, while player 1 receives 9 minus the demand of player 2. If player 1 rejects, then both players receive 0 . Show that it is a subgame-perfect Nash equilibrium outcome that player 1 demands 2 and that player 2 accepts! Are there any other subgame-perfect Nash equilibrium outcomes?

Consider the strategy for player 1 where 1 in the first period demands 2 and, after player 2 has rejected and made a demand in the second round, accepts any positive amount, while rejecting if player 2 demands 9, and the strategy for player 2, where 2 in the first round accepts if player 1 demands 0,1 or 2, while rejecting if player 1 demands more than 2, and demands 8 in the second round. The combination of these strategies determines Nash equilibria in all subgames, hence, it is a subgame-perfect Nash equilibrium. It is also a subgame-perfect Nash equilibrium if player 1 in the first period demands 1 and, after player

2 has rejected and made a demand in the second round, always accepts, and player 2 in the first round accepts if player 1 demands 0 or 1, while rejecting if player 1 demands more than 1, and demands 9 in the second round.

## Part B: First and second-price sealed-bid auction

Weight: $18 \%$ (with equal weight $=6 \%$ on each subproblem)
Consider a private value sealed-bid auction with two bidders who both are risk-neutral, have valuations that are independent and uniformly distributed on the interval $[0,10]$ and can submit bids in the interval $[0,10]$. The bidders are first informed of their valuations, they then submit their bids, and the item is assigned to the bidder with the highest bid (and with equal probability to each if the two bids are equal). In a first-price auction the winner pays the highest bid (i.e., their own bid), while a second-price auction the winner pays the second-highest bid (i.e., the bid of the other bidder. The utility of the winner equals their valuation minus the payment, while the other bidder has zero utility.
(i) Argue that it is a Bayesian Nash equilibrium to bid half of the true value in the first-price auction.

Calculations like those on pages 20 and 21 in the notes for the fifth lecture in game theory, show this result.
(ii) Argue that it is a Bayesian Nash equilibrium to bid the true value in the second-price auction.

It is a weakly dominant strategy to bid the true value.
(ii) Argue that the seller's expected revenue is the same for both types of auctions.

In the second-price auction, the winner pays the second-highest value. In the first-price auction, the winner pays the expected value of the second-highest value (the expected value of the other's value conditional on winning). In expectations, these payments are equal.

## Part C: Market for lemons

Weight: $18 \%$ (with equal weight $=6 \%$ on each subproblem)
Consider a market with 4 sellers and 6 buyers. The product can be of low, medium or high quality. Each buyer is only willing to pay a positive amount for one single unit of the product: 4.6 if of low quality, 8.8 if of medium quality, and 13.6 if of high quality. Each seller can produce at most two unit of a single quantity, with costs 1.4 for the first unit and 2.4 for the second unit if of low quality, with costs 4.6 for the first unit and 5.6 for the second unit if of medium quality, and with costs 11.0 for the first unit and 12.0 for the second unit if of high quality, This leads to the following demand and supply curves.


Seller first choose quality, then quantity, and finally price for each unit produced. Thereafter buyers decide whether to purchase available units.
(i) Assume first that the quality choices of sellers are observable for the buyers. Why is the equilibrium outcome that sellers choose medium quality, with two of them producing 2 units and two of them producing 1 units, with all 6 produced units selling at price 5.6?

Note first that there will always be profit opportunities for producing the medium quality if less than 6 units of medium quality are produced. Hence, only medium quality will be produced and only 6 units. Competition between the firms lower the price to the marginal cost $=5.6$.
(ii) Assume then that the quality choices of sellers are unobservable for the buyers. Why is the equilibrium outcome now that sellers choose low quality, with two of them producing 2 units and two of them producing 1 units, with all 6 produced units selling at price 2.4?

Note first that firms will always gain by switching to low quality, since quality is unobservable. Hence, only low quality will be produced and only 6 units. Competition between the firms lower the price to the marginal cost $=2.4$.
(iii) When comparing the equilibrium outcomes, what is the efficiency cost of not having complete information?

The seller earn in total 4 on the lower-cost first units, independently of whether quality choice is observable. However, buyer earn $8.8-5.6=3.2$ on each of the six units being traded, in total 19.2, if quality is observable, but only $4.0-2.4=1.6$ on each of the six units being traded, in total 9.6, if quality is unobservable. Hence, gains from trade is $4.0+19.2=23.2$ when quality is observable, but only $4.0+9.6=13.6$ when quality is unobservable.

